

The Philosophical and Mathematical Context of two Gerbert's Musical Letters to Constantine

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Abstract:

The paper deals with two letters written by Gerbert of Aurillac to Constantine of Fleury. In these letters Gerbert points out some passages from Boethius's *Introduction to Music* (II, 10; respectively IV, 2 and II, 21) concerning mathematical operations (multiplication and subtraction) with superparticular ratios i.e. ratios of the type $(n+1) : n$. The musical harmonies rule the Cosmos and the Celestial Spheres according to Martianus Capella *De nuptiis Philologiae et Mercurii*; Music is the basis for understanding Astronomy. This paper follows two main aims: philosophical importance of music as liberal art and mathematical basis of the Pythagorean tuning.

Keywords: music; philosophy; mathematics; Gerbert of Aurillac; Boethius

1. Introduction

While summarizing in one of his texts the Pythagorean teachings, Aristotle stresses out the importance of numbers and harmony for their ability to explicate the visible as well as non-visible reality: Numbers are the beginning of all things and represent basis of the individual entities and soul. They determine time as well as justice, delimit the course of this World and determine the principles of Cosmos. The numbers are mutually interconnected in ratios creating a harmony that penetrates the entire Universe. The harmony resonates through the music of the spheres that permeates and sounds in the entire Cosmos and together with the number it

represents fundamental metaphysical structure of the entire reality.¹ Therefore since the pre-Socrates Antiquity Music has been closely linked to the art of Mathematics, its importance doesn't lie only in its euphony and aesthetic-ethical effects but it rather represents the metaphysical order of reality that has the power to divert the human being from his sensual and changeable world and raise him closer to the divine origin.

Also Gerbert of Aurillac used his excellent knowledge of Music art as a significant stimulus for his individual career. In this sense Gerbert's biographer, friend, colleague and probably also pupil, Richer of Reims, informs us while describing Gerbert's visit in Rome in 970. Gerbert, as a member of the Catalan delegation, met the pope John XIII and later on also the emperor Otto I. Both of them were attracted by his knowledge of music and astronomy, as neither of these sciences was developed well in the Christian Europe (and also in Italy) of this time.² Music therefore represents Gerbert's stepping stone on his way to the emperor's court while his cooperation with the Ottonian dynasty in many ways shaped his further career (especially by obtaining the archbishopric and papal office).

In this article I will therefore target the Music theory as Gerbert described it in the course of his teaching during the end of the 70-ties of the 10th century in his two short texts – letters written to his close friend Constantine of Fleury who is

1 Aristotelés, *Metaphysica* I, 5, 985b. In Aristotelis *Opera omnia*. Ed. I. Bekkeri. Vol. I–V. Berlin: G. Reiner 1831–1870 (repr. Berlin: W. de Gruyter 1960).

2 Richerus Remensis, *Historiarum libri quattuor* III, 44. Ed. H. Hoffmann. *MGH SS*, t. 38. Hannover: Impensis Bibliopolii Hahniani 2000, p. 192.

in Gerbert's later letters prised as the great expert on Music.³

Both Gerbert's musical letters to Constantine are brief commentaries on selected passages of Boethius's *Introduction to Music*. Their author in fact doesn't deal with musical, philosophical or metaphysical overlapping contexts of the discussed topic. In both cases he strictly holds the mathematical explanation and theoretically as well as with help of practical examples he explains arithmetic operations with numeral ratios (musical intervals) or classifies results of the performed calculations. This approach totally corresponds with Gerbert's note written at the beginning of his tractate *Rogatus a pluribus*. He dissociate himself from aural (or generally empiric, based upon feelings raised by the Music) principles of the musical theory as presented by Aristoxenus of Tarentum and unambiguously wishes to follow the Pythagorean mathematical structure of the Music art including the tuning of organ tubes (*Mensura Fistularum*).⁴ As Gerbert deals in his musical letters exclusively with mathematics related to the Music science in my text I would like to stress out not only the mathematical context of his explanation but also to target the wider context of the philosophical consequences during the Early Middle Ages related to Music understood as science and art; and finally at

3 Gerbertus Remensis, *Epistola* 92. In Gerbert von Reims, *Die Briefsammlung Gerberts von Reims*. Ed. F. Weigle. MGH BDK, t. 2. Weimar: H. Bohlaus Nachfolger 1966, p. 121–122.

4 Gerbertus Remensis, *Rogatus* 1–8. In K.–J. Sachs, *Mensura fistularum. Die Mensurierung der Orgelpfeifen im Mittelalter*. T. 1: Edition der Texte. Stuttgart: Musikwissenschaftliche Verlag-Gesellschaft 1970, p. 59–60.

Sigismondi, C. *Gerberto e la misura delle canne d'organo*. In *Archivum Bobiense* 29 (2007), p. 355–396.

least but not last to briefly point out the importance of both letters for the musical theory.

2. Gerbert's Musical Letters to Constantine

One can assume that both Gerbert's commentaries on Boethius's *Introduction to Music* were written as answers to questions asked by Constantine. He probably during the end of the 70-ties of the 10th century left Reims where he previously studied and worked together with Gerbert and returned to his home monastery at Fleury. Gerbert's answers in both cases are of very similar structure: Firstly he turns our attention to a fragment of Boethius's text dealing with the relevant problem followed by a quotation from *Introduction to Music* accompanied by a brief explanation. Consequently Gerbert brings specific mathematical examples clarifying the quoted passage (or Boethius is mentioned again as a proof of the identical understanding of specific problem), whereupon the entire answer is generally summarized.

In case of the commentary on Boethius's *Introduction to Music* II, 10 and IV, 6 the question concerns doubling the superparticular ratios, i.e. ratios of type $(n+1) : n$, and the classification of the products. Boethius actually stated that the product of the superparticular ratio and number two doesn't result in the multiple or superparticular ratio.⁵ As an example Gerbert uses the sesquialter (i.e. perfect fifth or *diapente*, i.e. ratio 3 : 2). When the interval defined by the ratio 3 : 2 will be doubled it is obvious that the three-numerical sequence will be

5 Boethius, A. M. T. S., *De institutione musica* II, 10. In *Anicii Manlii Torquati Severini Boetii De institutione arithmetica libri duo: de institutione musica libri quinque*. Ed. G. Friedlein. Leipzig: Teubner 1867, p. 240: „...si superparticularis proportio binario multiplicetur id, quod fit, neque superparticulare esse, neque multiplex.“

created (for example numbers 4 – 6 – 9) while between the first and the second member in this sequence (numbers 4 – 6) the superparticular ratio is 3 : 2 (fifth). Equally between the second and the third member of the sequence (i.e. numbers 6 – 9) is the same superparticular ratio (i.e. second fifth). Two fifths, i.e. doubled fifth has for example form of numerical sequence 4 – 6 – 9. However between the first and the third number of this numerical sequence (i.e. numbers 4 – 9) is not a superparticular either multiplex ratio: It is a multiplex superparticular number, the ratio 9 : 4.⁶

In the second letter to Constantine Gerbert focuses on passage from the *Introduction to music* II, 21 where Boethius writes on two consequent superparticular ratios. When from the larger of the two superparticular ratios the lesser one (the immediately following one) is subtracted, the resulting difference of this subtraction will be lesser then one half of the subtracted ratio (the subtrahend) as the duplex product will be lesser then the subtrahend. Boethius offers specific example of subtracting a perfect fourth, i.e. *diatessaron* (ratio 4 : 3), from a perfect fifth, i.e. *diapente* (ratio 3 : 2). The result of this calculation is the whole tone that is a major second, i.e. *epogdous* or *tonus* (ratio 9 : 8). The duplex of the whole tone

6 Gerbertus, *Scholium ad Boethii Musicae Institutionis* l. II, c. 10; l. IV, c. 2. In *Gerberti postea Silvestri II papae Opera Mathematica* (972–1003). Ed. N. Bubnov. Berlin: R. Friedländer & Sohn 1899 (repr. Hildesheim: Georg Olms 1963), p. 29: „*Sit superparticularis proportio: quaternarius ad senarium. Haec quoniam una est, binario multiplicetur. Bis enim unum duo sunt. Oportet ergo, ut sunt IIII ad sex, sic esse sex ad alium quemlibet numerum; hic sit novem. Dico, quoniam novenarius ad quaternarium nec multiplex est nec superparticularis. [...] Quod ex priore multiplicatione natum est, duplex sesquiquartus est, ut sunt IX ad IIII, id est neque multiplex, neque superparticularis, sed multiplex superparticularis.*“

(ratio 81:64) is then a semitone lesser then a fourth (ratio 4 : 3).⁷ Gerbert demonstrates these relations by example of the smallest numerical sequence that expresses a ratio between a fourth and fifth: 6 – 8 – 9. From a fifth (i.e. interval *diapente* defined by numbers 6 – 9) he subtracts a fourth (i.e. interval *diatessaron* defined by numbers 6 – 8). The result is an interval between numbers 8 – 9, i.e. major whole tone (*tonus*). The double of this tone can be expressed for example by the three-member numerical sequence 64 – 72 – 81. While here (just like in the first Gerbert's letter) the first and the second member of this sequence (and alike its second and third member) have in between the very same ratio (here 9 : 8). The ratio between the first and the third member of this sequence (i.e. between numbers 64 and 81) is lesser than ratio 4 : 3. The obvious conclusion of this is that the ratio 9 : 8 is lesser then half of the 4 : 3 ratio, that is – as Gerbert notices – valid for all results of subtraction of two consequent superparticular ratios.⁸ This

7 Boethius, *De institutione musica* II, 21, p. 254: „*Ab omni superparticulari, si continuam ei superparticularem quis auferat proportionem, quae est scilicet minor, id, quod relinquitur, minus est ejus medietate, quae detracta est, proportionis: ut in sesquialtera vel sesquitertia. Quoniam sesquialtera major est, sesquiterciam de sesquialtera detrahimus. Relinquitur sesquioctava proportio, quae duplicata non efficit integram sesquiterciam proportionem, sed ea distantia minor est, quae in semitonio repperitur. Quod si duplicata sesquioctava comparatio non est integra sesquitertia, simplex sesquioctava non est sesquiterciae proportionis plena medietas.*“

8 Gerbertus, *Scholium ad Boethii Musicae Institutionis l. II, c. 21*, p. 31: „*Sit propositus unus et idem numerus, ad quem aptetur sesquialtera ac sesquitertia proportio: hic sit VI, ad quem VIII sesquialter est, VIII vero sesquitercius. Qui disponantur hoc modo: VI, VIII, VIII. Et quoniam hae duae proportiones continuae superparticulares sunt in tribus terminis constitutae: VI, VIII, VIII, auferamus primum terminum, ad quem VIII est sesquitercius, — VIII sesquialter. Remanet VIII et VIII, qui sesquioctavi sunt. Sed sesquioctava porportio non est medietas*

statement is in his letter supported by two schemes demonstrating this rule on the results of subtracting a fourth from a fifth and the ratio 5 : 4 from the ratio 4 : 3.⁹

3. Music as a Mathematical Science – Arithmetic Operations with Ratios

As Gerbert follows Boethius (and Pythagorean) mathematical fundament of the musical theory, it is natural that his musical letters mainly have a character of brief mathematical tractate presenting selected topics related to arithmetic operations with fractions (ratios). Although they seem at the first sight to deal with demanding computations, there are basically elementary operations.

In the first of his letters Gerbert speaks about doubling the ratios and as an example he presents the superparticular number 3 : 2. Mathematically we can express Gerbert's example in the following way:

$$[i] \quad 2 \cdot (3 : 2) = 2 \cdot (3 / 2) = 3^2 / 2^2 = 9 / 4 = 9 : 4.$$

From this calculation we can easily deduct that the result of a fifth doubling doesn't represent the multiplex ratio (i.e. example, when the lesser number is contained in the greater

minoris proportionis, id est sesquiertiae, quoniam duplicata non efficit eam, sed minor est. Duplicemus igitur sesquioctavam proportionem et sint tres numeri ita dispositi, qui a proportione VIII et VIII non recedant, fiantque octies octo et VIII VIII et VIII VIII, id est; LXIII, LXXII, LXXXI. Dico quoniam primus ad secundum et secundus ad tertium sesquioctavam custodiunt habitudinem, sed tertius ad primum minus est, quam sesquiertius, non est ergo sesquioctavus medietas sesquiertii.“

9 Gerberti postea Silvestri II papae Opera Mathematica, Fig. 2.

one more then once and fully corresponds with the value of the greater number; for example ratios 2 : 1, 3 : 1, 4 : 1 and so on; therefore double, triple, quadruple etc.) nor the superparticular ratio (i.e. case, when the lesser number is contained in the greater number once but there is a certain part of the lesser number remaining till the full value of the greater number; the remaining part can be expressed as a unit fraction of the lesser number, i.e. fraction where the number one is the numerator; for example ratios 3 : 2, 4 : 3, 9 : 8 and so on, i.e. *diapente*, *diatessaron*, *tonus* etc.), but it is the multiplex superparticular (i.e. number, when the lesser number is contained in the greater one more then once and till the full value of the greater number remains certain part of the lesser number expressed by the unit fraction, i.e. fraction where the number one is a numerator; for example ratios 5 : 2, 7 : 3, 9 : 4 and so on).¹⁰

If we wish to express the entire calculation as an interval visualized on the numeral axis, it can be demonstrated for example in this way (fig. 1):

10 See for example Nicomachos, *Introductionis Arithmeticae* I, 22, p. 59–63; Boethius, *De institutione arithmetica* I, 29, p. 73–76; Martianus Capella, *De nuptiis Philologiae et Mercurii* VII, 763, p. 281–282; Cassiodorus, *Institutiones* II, 4, p. 137–138; Isidorus Hispalensis, *Etymologiae* III, 6, p. 292; Abbo of Fleury, *Commentary on the Calculus*, p. 119–120 etc.

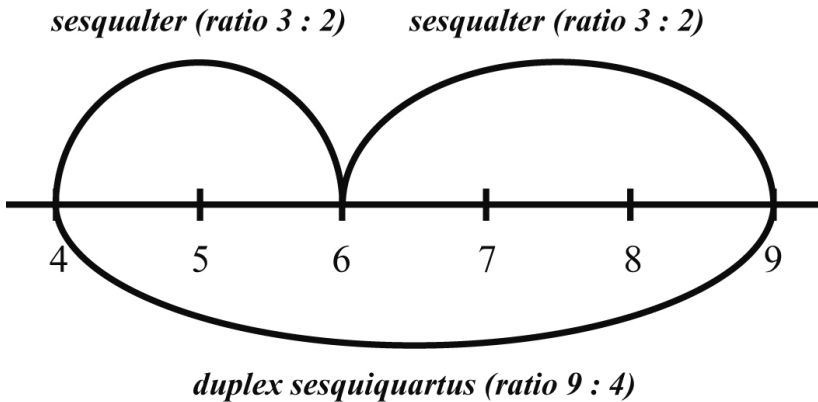


Fig. 1 – Duplex of the superparticular ratio 3 : 2 gives result of the multiplex superparticular ratio 9 : 4; based on the Gerbert's example given in his letter to Constantine

Result of the doubled fifth is ratio 9 : 4 therefore it is not the multiplex either a superparticular number, but it is the multiplex superparticular. It is necessary to note that the multiplex superparticular isn't result of doubling of any superparticular numbers but only of a fifth. In case of doubling other superparticular ratios (for example fourth, sesquialter, whole tone, i.e. major second, and so on) the result is always the superpartient ratio (i.e. relation between two numbers while the lesser number is contained in the greater number and in order to reach the value of the greater number it is necessary to add a certain proportion of the lesser number that cannot be expressed by a unit fraction of the lesser number).¹¹ So it is that the doubled fourth (4 : 3) stands for a

11 See for example Nicomachos, *Introductionis Arithmeticae* I, 20, p. 55–58; Boethius, *De institutione arithmetica* I, 28, p. 70–73; Martianus Capella, *De nuptiis Philologiae et Mercurii* VII, 762, p. 280; Cassiodorus, *Institutiones* II, 4, p. 137; Isidorus Hispalensis,

minor seventh (16 : 9) and doubled major second stands for a major third (81 : 64) and so on.

While Gerbert doesn't deal in his letter with the importance of multiplication of a fifth,¹² it is possible to assume that Constantine, the addressee of his letter, was aware of it. Gerbert therefore only reacts to the arithmetical part of the problem as the musical one seems to be clear. What is then the importance of the double fifth (alternatively its multiple or division) in the music theory? It is a fifth and its powers standing behind the Pythagorean fundament of mathematically expressible tuning based on the mutual relations between an octave (*diapason*, ratio 2 : 1) and a fifth (ratio 3 : 2). When there are defined relative relations between the tones within an octave (in modern way expressed for example from C_1 to C_2) the multiplication and division by a fifth and an octave is used. The fifth ratio (*diapente*, 3 : 2) can be obtained by multiplication of a perfect prime (1 : 1, in the basic major scale the C tone (Do); see [ii]) while we obtain the G (Sol) tone (so called the perfect fifth):

$$[ii] \quad (1 : 1) \cdot (3 : 2) = (1 / 1) \cdot (3 / 2) = (1 \cdot 3) / (1 \cdot 2) = 3 / 2 = 3 : 2.$$

Etymologiae III, 6, p. 292; Abbo of Fleury, *Commentary on the Calculus*, p. 118–119 etc.

12 On Gerbert's teaching method in the field of music see Richerus, *Historia* III, 49, p. 195. Furthermore see for example C. Sigismondi, *Gerberto e la misura delle canne d'organo*. In *Archivum Bobiense* 29 (2007), p. 355–396; F. G. Nuvolone, 'Gerbertus musicus' e le attività culturali bobbiesi dell'annata 2002. In *Archivum Bobiense* 24 (2002), p. 7–48; M. Huglo, *Gerbert, théoricien de la musique, vu de l'an 2000*. In *Cahiers de civilisation médiévale* 43/170 (2000), p. 143–160; K.–J. Sachs, *Gerbertus cognomento musicus. Zur musikgeschichtlichen Stellung des Gerbert von Reims (nachmaligen Papstes Silvester II)*. In *Archiv für Musikwissenschaft* 29/4 (1972), p. 257–274 etc.

The perfect fourth (*diatessaron*, ratio 4:3) can be mathematically obtained by dividing a prime by a fifth (the resulting tone would be one fifth lesser then the basic tone; see [iii_a]) and consequently this proportion multiplied by an octave (ratio 2 : 1; see [iii_b]) the *F* tone (Fa) is obtained:

$$[\text{iii}_a] \quad (1 : 1) / (3 : 2) = (1 / 1) / (3 / 2) = (1 \cdot 2) / (1 \cdot 3) = 2 / 3 = 2 : 3;$$

$$[\text{iii}_b] \quad (2 : 3) \cdot (2 : 1) = (2 / 3) \cdot (2 / 1) = (2 \cdot 2) / (3 \cdot 1) = 4 / 3 = 4 : 3.$$

Major second (a major whole tone; i.e. *epogdous* or *tonus*) is defined as the doubled fifth (see [iv_a]) divided by an octave (*diapason*; see [iv_b]) while at the same time the *D* tone (Re) is obtained:

$$[\text{iv}_a = \text{i}] \quad 2 \cdot (3:2) = 2 \cdot (3/2) = 3^2/2^2 = 9/4 = 9:4;$$

$$[\text{iv}_b] \quad (9:4)/(2:1) = (9/4)/(2/1) = (9/1)/(4/2) = 9/8=9:8.$$

In the similar way a major third (*E* tone, Mi), a major sixth (*A* tone, La) and a major seventh (*H* tone, Si) can be obtained. It is now obvious why did Constantine ask Gerbert to explain the way of double the superparticular ratios as their mutual multiplication and division of ratio 2 : 1 and ratio 3 : 2 allows to define the tone scale in full. Also the second Gerbert's music letter deals almost exclusively with the arithmetic operations with numeral ratios, i.e. music intervals. In this case it deals with subtraction and multiplying by two. Gerbert's example shows that the result of subtraction of the two consequent superparticular ratios (see [v]) is lesser then one half of the subtracted ratio as the doubled result of this subtraction (see [vi]) is lesser then the entire subtracted ratio (see [vii] and [viii]). Modern mathematical notation expresses it for example in this way:

$$[\text{v}] \quad (3 : 2) - (4 : 3) = (3 / 2) - (4 / 3) = (3 / 2) \cdot (3 / 4) = (3 \cdot 3) / (2 \cdot 4) = 9 / 8 = 9 : 8$$

[vi] $2 \cdot (9 : 8) = 9^2 / 8^2 = 81 / 64 = 81 : 64$

[vii] $(4 / 3) - (81 : 64) = (4 / 3) \cdot (64 / 81) = 256 / 243 = 256 : 243$

[viii] $81 : 64 < 4 : 3$

As expressed in musical terms: If we subtract a fourth from a fifth, a major second (major whole tone) is obtained and two major seconds (two whole tones) are one *limma* lesser then a fourth. From the mathematical point of view it basically means that if we subtract from ratio 3 : 2 (for example interval determined by numbers 6 – 9 or 192 – 288) the ratio 4 : 3 (for example interval determined by numbers 6 – 8 or 192 – 256) the result will be the ratio 9 : 8 (for example interval determined by numbers 8 – 9 or 256 – 288). If the ratio 9 : 8 is doubled we obtain the ratio 81 : 64 (for example numeral sequence 64 – 72 – 81 or 192 – 216 – 243). The ratio 81 : 64, i.e. double of the ratio 9 : 8 is then lesser then the ratio 4 : 3 (for example interval determined by numbers 63 – 84 or by numbers 192 – 256) by the ratio 256 : 243 as more illustratively demonstrated in the following diagram (fig. 2).

Also in this case Gerbert doesn't bother to stress out the importance of presented calculations for the musical theory itself. The size of whole major tone interval (major second, i.e. ratio 9 : 8) can be (as demonstrated on the Gerbert's first letter) obtained with help of multiplication and division of a fifth by an octave but it can also be defined with help of subtraction of a fourth from a fifth. This way we can also define the firm mutual relation between a fifth and a fourth, or between a fourth and a major second. Fig. 2 demonstrates that while a fifth represents a sum of a fourth and a whole major tone, a fourth is a sum of two whole tones and one semitone.

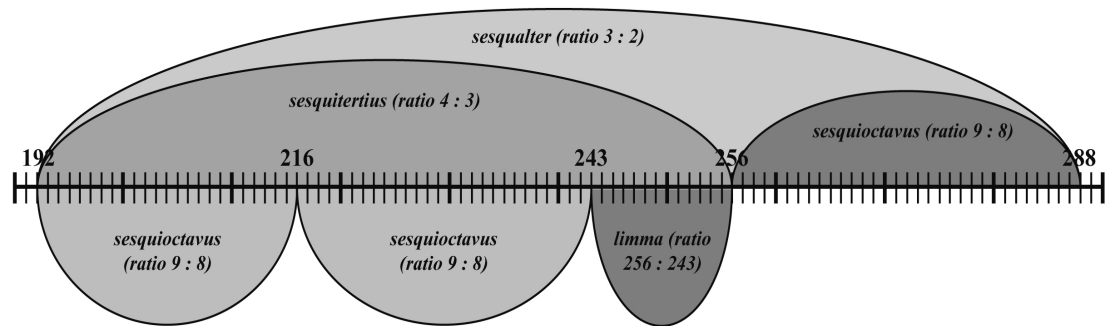


Fig. 2 – Duplex of the difference between the fifth and fourth is lesser then the fourth; based on the Gerbert's example from his letter to Constantine

The conclusion is: As an octave is a sum of a fifth and a fourth (see [ix]), within one octave the distances between the individual tones (C_1 to C_2) are determined by the intervals of five tones (ratios 9 : 8) and two semitones (between tones e and f and between tones h and c_2 ; in other cases there are intervals of the whole tones). The mathematical formula of these musical relations can be presented for example in the following way:

$$[ix] \quad (2 / 1) = (4 / 3) + (3 / 2) = (4 \cdot 3) / (3 \cdot 2) = 12 / 6 = 2 : 1.$$

$$[x] \quad (4 / 3) = (9 / 8) + (9 / 8) + (256 / 243) = (9 \cdot 9 \cdot 256) / (8 \cdot 8 \cdot 243) = 20\,736 / 15\,552 = 4 : 3.$$

$$[xi] \quad (3 / 2) = (4 / 3) + (9 / 8) = (4 \cdot 9) / (3 \cdot 8) = 36 / 24 = 3 : 2.$$

$$[xii] \quad (2 / 1) = 5 \cdot (9 / 8) + 2 \cdot (256 / 243) = (9^5 / 8^5) + (256^2 / 243^2) \\ = (59\,049 / 32\,768) + (65\,536 / 59\,049) = (59\,049 \cdot 65\,536) / (32\,768 \cdot 59\,049) = 3\,869\,835\,264 / 1\,934\,917\,632 = 2 : 1.$$

This problem also plays very important role in the musical theory as it defines tones (sounds) of an octave with help of the whole tones and semitones.

If we decide to follow the Gerbert's (and Boethius's) example

and subtract another two consequent superparticular ratios, i.e. ratio 5:4 from ratio 4:3 the result will be an interval 16:15 (i.e. lesser diatonic semitone). Its doubled quantity will provide ratio of two semitones or of so called diminished third (ratio 256:225). Differences of other consequent superparticular ratios thus also provide various small intervals also used in the music theory.

4. Music and its Relation with Knowledge and Philosophy

Music with its theory whether based on the aural perception or on the mathematical fundamentals was considered in the Middle Ages as a part of the liberal arts. Although Gerbert in his letters to Constantine remains silent about the reason for his interest in Music or about the importance of this art, in the following part of the text I will address more general context of the Music art and knowledge thus focus the Music as such and its position within the (early) medieval struggle for wisdom.

Traditional definition of Music as known in the Middle Ages from the Late Antiquity and explicitly defined in for example Augustine's tract *De Musica*, is closely tied to a melody creating, i.e. knowledge of the proper measure.¹³ During the early Middle Ages we often come across this definition perhaps in small variations.¹⁴ In his dialogue *De Ordine*

13 Aurelius Augustinus, *De musica* I, II, 2. In Aurelius Augustinus, *De musica. Bücher I und VI. Vom ästhetischen Urteil zur metaphysischen Erkenntnis*. Hamburg: F. Meiner Verlag 2002, p. 6: „*Musica est scientia bene modulandi*.“ Furthermore see Censorinus, *De die natali liber* 10, 3. Ed. N. Sallmann. Leipzig: Teubner 1983, p. 16.

14 See for example Martianus Capella, *De nuptiis Philologiae et Mercurii*

Augustine explains in more details the systematic taxonomy of liberal arts. Music he mentions on the first place among the mathematical sciences. This prestigious status of music is well deserved as music represents a certain form of transition between the sensory perception and the purely rational cognition. The aural perception holds the irreplaceable position within the Music art as it naturally informs us about sounds, tones, melodies, rhythm etc. Sensory impulses can cause pleasure or displeasure however various consonances and harmonies are neither set by the hearing itself but by a number related to another number (i.e. creating the ratios). Therefore a number is always and necessarily the fundament of music in any of its forms. As a number is of God's origin it is not possible to doubt the divine origin of music. Its manifestations proof the harmony and divine perfection that created and organized this world.¹⁵

Unlike Augustine Martianus Capella in his tractate *De nuptiis Philologiae et Mercurii* positions Music as the very last one of all liberal arts. Music represents a certain form of climax of all knowledge. Each of the liberal arts in Martianus's description can be assumed by the appearance of corresponding sister-Muse attending at the wedding gathering of gods in order to present the fundamental features of her art. The arrival and explanation of Harmony (Music) is in some aspects unique. Harmony is awaiting her entrance for the longest time of all *artes*, gods are arguing for a long time whether they actually want to hear something else (as they would rather start the

IX, 930, p. 356; Cassiodorus, *Institutiones* II, 5, p. 143; Isidorus Hispalensis, *Etymologiae* III, 15 etc.

15 Aurelius Augustinus, *De ordine liber duo* II, 14, 39–41. Ed. W. M. Green. CCSL 29. Turnhout: Brepols 1970.

wedding feast) and more the entry of the seventh sister is even precluded by the refusal of another two sisters-Muses: Architecture and Medicine.¹⁶ Despite the gods arguing whether to listen another lecture on the liberal art or not, the entrée of Harmony ends all argument and disputes. The presence of this art of this own (naturally accompanied by strong visual and aural effects) calms and astonishes everyone.¹⁷ This harmonic and positive atmosphere persists during Harmony's presentation of the musical art¹⁸ envied by another sisters. Harmony astonishes everyone by her noble appearance and delicious rustling sound of her robe as well as by sounds of her musical instruments including strange round shield interlaced with strings. The gods are so enchanted by Harmony and her Music that they fully recognize the divine character of her performance and soon they show her great respect and even stand in ovation.¹⁹ Martianus Capella presents Music as superior to gods – they honour her, she calms them with her melodies, brings them a rest, prudence, harmony and perfection. Music is therefore reaching the ultimate heights.

Similarly Isidor of Sevilla in his *Etymologies* states that no knowledge (*disciplina*) is perfect without Music, as without music nothing would exist.²⁰ This is how the Pythagorean idea

16 Martianus Capella, *De nuptiis Philologiae et Mercurii* IX, 888–898, p. 337–342.

17 Martianus Capella, *De nuptiis Philologiae et Mercurii* IX, 905, p. 334–345.

18 Martianus Capella, *De nuptiis Philologiae et Mercurii* IX, 996, p. 384.

19 Martianus Capella, *De nuptiis Philologiae et Mercurii* IX, 910, p. 347.

20 Isidorus Hispalensis, *Etymologiae* III, 17.

on importance of Music disseminated across the Europe – the entire Universe and Cosmos were created with help of tonal scales and everything celestial is moving accompanied by sound of cosmic harmony.²¹ Cosmological and metaphysical fundament of the Universe is determined by Music (i.e. by numeral ratios). These numbers allow us to recognize the truth about Cosmos and at the same time they reach human and his soul. The sensory perception is adjusted by the rational cognition of numeral relations in man's soul.

Also Boethius agrees with Augustine that Music approaches us through the hearings however it is the harmony that can engage the rational part of the human soul. It is the harmony that is reflected by the entire behaviour of human personality and that essentially effects the ethical activities of human being. Therefore Music forms the character of man however behind the misleading and unstable sensory (aural) cognition it is always necessary to seek for true, stable and permanent proportional relations mathematically defining the proper value of Music.²²

The mathematical ground of Music, typical for Middle Ages as shown for example in *Lucidarium* by Marchetto of Padua,²³

21 Isidorus Hispalensis, *Etymologiae* III, 17: „*Nam et ipse mundus quadam harmonia sonorum fertur esse conpositus et caelum ipsud sub harmoniae modulatione revolvi.*“

22 Boethius, *De institutione musica* I, 1, p. 179–187.

23 Jan W. Herlinger, *The Lucidarium of Marchetto of Padua. A Critical Edition, Translation, and Commentary*. Chicago: University of Chicago 1985, p. 82–84. Furthermore see M. Celhoffer, *Hudba, animus a proporce v kontextu Marchettova Lucidaria aneb „svůdné vábení Sirén“*. In *Musicologica Brunensia* 46/1–2 (2011), p. 49–54.

probably stands behind the Gerbert's commentaries of Boethius's *Introduction to Music*: If it is the mathematical proportional ground that forms the order of Universe, Cosmos and humankind including his way towards the Good and the Truth represents the determining aspect of Music (mainly in case of so called *Musica mundana* and *Musica humana* if Boethius's terms would be used²⁴) then naturally the mathematical aspect of Music becomes the principal that needs to be discovered if we decide to perceive music as a science and a knowledge rather than as "only" a practical performance of musical art.

5. Conclusion

The Music in Middle Ages was regarded as a liberal art, i.e. knowledge that should lead us to the Truth and more it doesn't deal only with Music perceived by hearing (for example the Music of the spheres that is not audible²⁵) but it fundamentally forms the very structure of the Universe. The Music stood at its beginnings and it defines rules of this World (by the numeral ratios, i.e. music intervals) in the similar way as it forms a human being himself to whom it opens the way to perfection in cognition as well as in all his actions. As rather the obvious result of Music art perceived in this way it can also allow us to concentrate primarily on the mathematical description of musical fundamental relations.

Gerbert's commentary on Boethius's *Introduction to Music* follows the above-mentioned approach as he was probably

24 Boethius, *De institutione musica* I, 1, p. 187–189.

25 Boethius, *De institutione musica* I, 9, p. 196.

answering questions raised by Constantine of Fleury. Gerbert's typical emphasis on the pedagogical exemplary explanation and at the same time on the practical usability of certain intellectual problem led him to the effort of explaining the musical (and also metaphysical and cosmological) problems with help of the most simple examples and models using his excellent mathematical erudition to the best of his ability.

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