

THE PHYSICS OF CLOUDS

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Preface to the Second Edition

THIS book has the same structure and scope as the first edition, but the text has been completely re-written, revised, and enlarged to accommodate the significant advances that have taken place during the intervening decade. It attempts to provide a fairly complete and critical account of all the important developments up to the end of 1969 and includes some important work published in 1970.

Again the emphasis is on the microphysical processes of condensation of water vapour to form droplets, the supercooling, nucleation, and freezing of water droplets, the growth and aggregation of snow crystals, the mechanisms of raindrop, snowflake, and hailstone formation, the radar detection of precipitation elements, and the various processes of cloud electrification, with only sufficient background information on the structure and dynamics of cloud systems to allow realistic discussion of the particle physics. However, in the preceding preface, written thirteen years ago, reference was made to the strong interactions that exist between these microphysical processes and the air motions in and around clouds, and to the importance of acquiring a much deeper understanding of the cloud dynamics for the development of the subject as a whole. The hopes expressed then for much greater effort and progress in this direction, and for the appearance of a major text on this aspect of the subject, have yet to be realized, but there is now a much greater awareness of the need. For, while considerable progress has been made in establishing the physical laws that govern the nucleation, growth, and aggregation of particles, extrapolation of the knowledge obtained under controlled laboratory conditions to the evolution of populations of particles in the vastly more complex environment of a natural cloud requires great caution while our understanding of cloud dynamics is so rudimentary. Indeed it may be difficult to identify the important gaps in our current understanding of the microphysical events until this has been formulated in a dynamical context and tested against the results of observation, measurement, and prediction. The use, in recent years, of new, especially Doppler, radar techniques, of modern methods of data processing, and of numerical models, has given the subject a new impetus in this direction. I hope that this may lead, in the next decade, to advances comparable to those made in the laboratory during the 1950s and 1960s and which form the basis of this book.

Again, I am grateful to the learned societies, publishers, and authors, acknowledged in the text, for permission to reproduce many of the diagrams.

I am indebted to Miss Heather May and Mrs Janet Bolton for their help in preparing diagrams and an earlier typescript of the text, and especially to my present secretary, Miss Eileen Forde, for her invaluable assistance in producing the final manuscript, the index, and the bibliography. Lastly, I wish to acknowledge the skill and care which the Clarendon Press have exercised in the production of the book.

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B. J. MASON

Preface to the First Edition

CLOUD physics is concerned with those processes which are responsible for the formation of clouds and the release of precipitation. The subject has expanded enormously during the last decade and is now one of the most flourishing branches of meteorological physics. During this short period it has developed from the pursuit of a few individual scientists with modest resources into a group activity requiring elaborately instrumented aircraft, radar, and a wide range of laboratory facilities. This rapid development along a broad front has produced a large and diverse literature, the growth of which has made life increasingly difficult for the research worker and impossible for the student. Accordingly, it appeared that an account of the present state of the subject might serve a useful purpose although the disadvantages of writing a book at a time when the field is rapidly growing have become only too apparent. In this volume I have attempted to give a fairly comprehensive account of recent researches, both experimental and theoretical, on the micro-physical processes of nucleation, condensation, droplet growth, the initiation and growth of ice crystals, and the mechanisms of precipitation release. There follow also discussions on the present status of rain-making experiments, which have stimulated much of the fundamental research; on radar studies of precipitating clouds; and on the electrification of clouds which, I feel, should be regarded as an integral part of cloud physics.

Although the emphasis here is upon the *micro-physical* processes, it is important to recognize that these are largely controlled by the atmospheric motions which are manifest in clouds. These *macro-physical* features of cloud formation and growth, which might more properly be called a *dynamics*, provide a framework of environmental conditions confining the rates and duration of the microphysical events. For example, the growth or freezing of cloud droplets is accompanied by the release of great quantities of latent heat, profoundly influencing the motion of cloudy air masses, while the motions which ultimately cause evaporation of the cloud determine its duration, and will set a limit to the size which its particles can attain. Progress in cloud physics has been hindered by a poor appreciation of these interrelations between processes ranging from nucleation phenomena on the molecular scale to the dynamics of extensive cloud systems on the scale of hundreds or thousands of kilometres.

Unfortunately, our present understanding of the large-scale physics of clouds is rudimentary. This is partly because this aspect of the

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subject has not received the attention it deserves, but mainly because of the difficulty of obtaining observational information about air motions on a scale too large to be simulated in the laboratory and yet too small to be defined by the observational network used in yet forecasting. Because I am convinced that future progress will be largely governed by our improved understanding of cloud dynamics, I hope that the next few years will see a greatly increased effort in this direction. At this stage it seems advisable to stress only those aspects which are firmly based upon observation and which are necessary to provide an adequate background for discussion of the micro-physical processes. Cloud dynamics will, I hope, eventually form the subject of a separate volume.

In attempting to write a coherent and integrated account of the subject as I see it at present, I have aimed at being critical rather than exhaustive, and consequently the treatment reflects, to some extent, my personal views. In a new and rapidly growing field there is, of course, room for differences of opinion and of interpretation; it is difficult and perhaps undesirable to remain dispassionate in the heat of the battle. However, I have done my best to provide an up-to-date review of the subject and to point out those gaps in our knowledge which appear to merit urgent attention. I shall be pleased if it proves to be of some use to my fellow research workers, and if it draws the attention of the student to a field which offers enormous scope for research into a wide range of phenomena, both in the free atmosphere and in the laboratory, and by theoretical methods. I hope also that I have managed to convey a little of the continual pleasure and excitement which I have experienced while working in this field during the last few years.

In the preparation of this book I have received help from a number of people to whom I should like to express my grateful thanks. To my colleague, Mr. F. H. Ludlam, I am greatly indebted not only for writing the Introduction on 'The Large-scale Physics of Clouds' but also for many stimulating discussions on all aspects of the subject during the last seven years. I am very grateful to those friends who, by sending me their papers in advance of publication, have ensured that this book will appear only a few months out of date. For permission to reproduce many of the diagrams I am indebted to those learned societies, publishers, and authors acknowledged in the text. I am particularly indebted to Mrs. S. M. Devers, Miss E. M. Lea, Miss M. L. J. Pinhard, Mrs. M. Brookfield, and Miss S. A. Latta for their invaluable assistance in preparing the typescript and diagrams, and to the Clarendon Press for the skill and care which they have exercised in the production of the book. Lastly, I owe much to my

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wife who typed a good deal of the manuscript, and without whose continual help and encouragement this book would not have been written.

B. J. M.

*Imperial College, London
February 1957*

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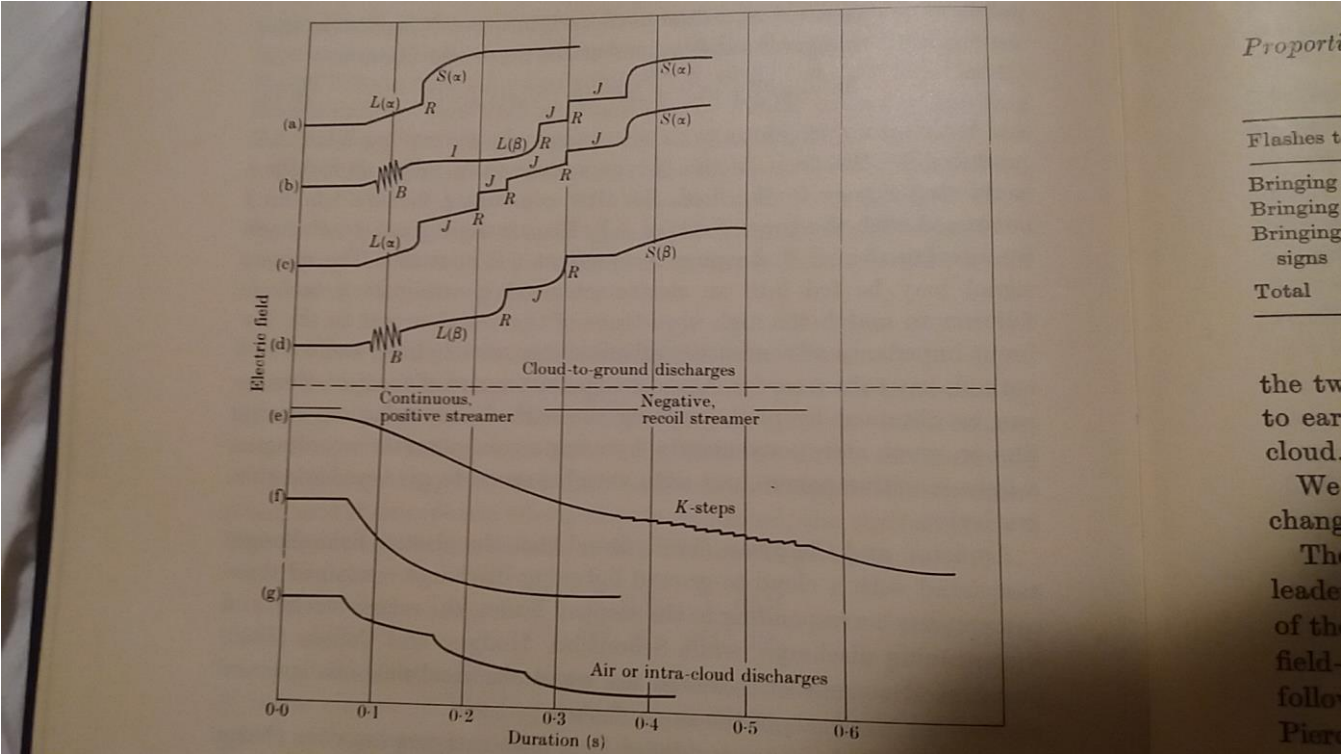
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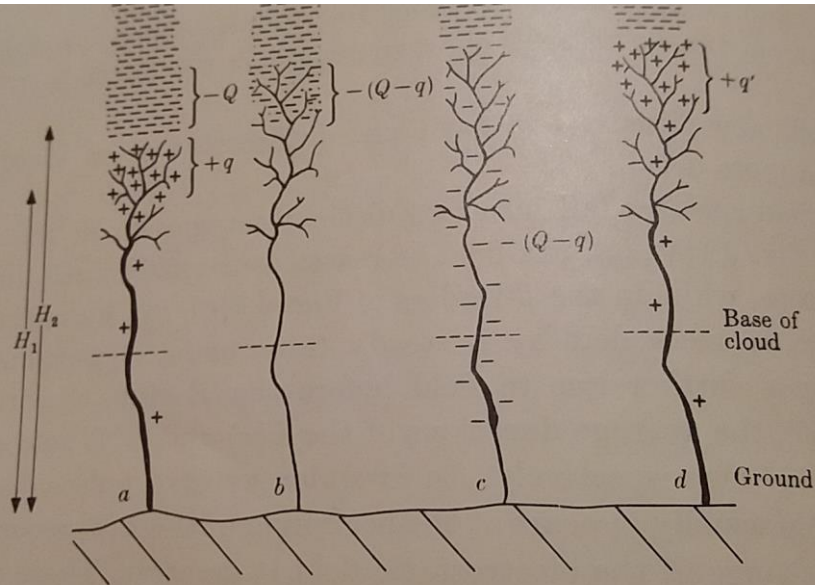


FIG. 9.8. Diagram illustrating the electrical processes taking place in the cloud during the interval between two successive strokes of a lightning flash to earth. (From Malan and Schonland (1951a).)

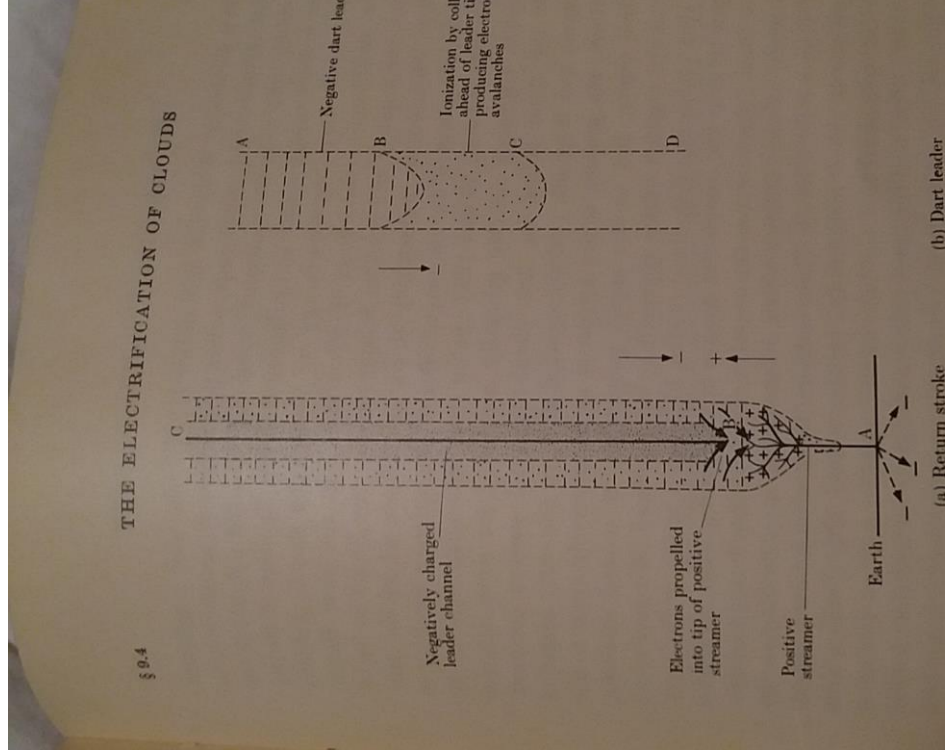


FIG 9.9. The mechanism of (a) the return stroke, (b) the dart leader. (After Schonland.)

of electrons in the critical field. This explains why dart leaders velocities as high as 10^8 – 10^9 cm s⁻¹, the higher values occurring the time interval since the previous return stroke is small, so that density of free electrons in the channel has not appreciably decreased recombination or attachment to air molecules.

9.4. The electrical structure of the thunderstorm

We have already seen how Wilson deduced the bipolar structure of the thunderstorm. The diagram shows the electrical charges situated above

the pilot channel must be thermally ionized, which requires a minimum field of 60 kV cm^{-1} . The large-scale field below a thunderstorm is much less than this, perhaps only a few thousand V/cm , but the non-uniform field near the tip of the channel may well exceed 60 kV cm^{-1} for some distance ahead. The pilot streamer may therefore advance a limited distance to a point at which the field drops below the critical value. A sudden change from glow to arc conditions is now believed to take place in the newly-formed channel, which becomes heavily ionized and appears as a bright step. The whole channel from the cloud to the tip of the step is now highly conducting, and with the tip at much the same potential as the cloud, the strong field between it and the surrounding air is re-established, lateral corona currents develop, and the pilot leader now advances a further step. Different theories of the mechanism of the sudden transformation from glow to arc conditions in the channel have been proposed by various authors, notably Schonland (1938) and Beane (1944) but, being largely based on the behaviour of discharges between solid electrodes, it is not certain which of these most nearly describes what happens in a lightning flash.

The return stroke, illustrated in Fig. 9.9(a), is a positive streamer, AB , advancing from the ground upwards into the ionized and negatively-charged channel of the leader stroke BC . In the vicinity of B , at the junction of the positive streamer and the highly-charged negative channel, the intense electric field will drive free electrons at great speed into the advancing positive tip. As they move downwards, these electrons will create electron avalanches, ionize and heat the air in their path, and leave behind a highly conducting channel. For these reasons, the positive streamer moves upwards at extremely high velocity, even though the positive ions are themselves relatively immobile. As it moves upwards, it passes to ground the electrons in both the stem and branches of the leader, and finally discharges a negative region of the cloud.

The dart leader is a negative streamer advancing along a previously ionized channel, so that the region in front of the advancing tip contains a high concentration of free electrons. In Fig. 9.9(b), AB is the negative dart leader, BCD the previously ionized channel in front of the tip B , and BC a region in which the electric field is strong enough to produce ionization by collision. The free electrons originally present in BC move forward away from B and produce electron avalanches, make the air highly conducting, and effectively extend the streamer to C . The streamer may be regarded as jumping forward from one avalanche to another at a velocity which may be much higher than the drift velocity

9.3.3. *The mechanisms of electrical breakdown*

As pointed out on p. 503, the fact that the stepped leader, travelling at its minimum velocity of 10^7 cm s⁻¹, would travel from cloud base at 2 km to ground in 20 ns, while the associated electric field changes frequently fast for 50–150 ns, and the appearance on the early part of the record of radiation pulses at much longer intervals than those associated with the bright steps of the leader, suggest that the stepped leader is preceded by an initial breakdown process of very different nature. Malan and Schenckland (1951a, b) have obtained evidence to suggest that a high proportion of the β leaders originate near the 0°C level, in the region of strong field existing between the base of the main negative column and the subsidiary positive charge located near the cloud base that is generally associated with heavy rain. Now Mackay (1931) found that, under the influence of strong electric fields, water droplets became elongated in the direction of the field, and when this exceeds a value $R'_0 = 3875/\sqrt{R}$ V cm⁻¹, where R is the drop radius in cm, they become unstable, and streamers develop at the ends with the onset of corona discharge. Thus, in the presence of drops of $R = 1$ mm, streamers would develop in fields exceeding $12\,000$ V cm⁻¹, the critical field for 2 mm drops being 8700 V cm⁻¹—much less than the minimum value of 30 kV cm⁻¹ required to initiate the breakdown of dry air. The high field need only be very localized because a streamer starting from one drop may propagate itself from drop to drop under a much weaker field. The breakdown (B) streamers now establish a conducting path between the main negative column and the base of the cloud, the lower positive charge gets neutralized, and the streamer channels become progressively more negatively charged. This is indicated by a slow steady change (intermediate- or I -change) on the electric-field record. The large radiation pulses accompanying the B process are probably caused by separate streamers starting in succession in the region just above cloud base.

The *stepped leader* is a negative streamer advancing in step-wise fashion at an overall velocity similar to that of a negative streamer of a laboratory spark in air. This similarity suggests that each bright step-like advance is preceded by a pilot streamer rather like that of the laboratory spark. Laboratory experiments have shown that a minimum field of 30 kV cm⁻¹ is required to propagate a negative streamer by an electron avalanche in which free electrons ionize the air molecules by collision, the drift velocity of electrons in this field being 10^7 cm s⁻¹. In addition, it appears that, for the development of the leader process,

Some Useful Physical Constants

Universal constants

Avogadro's number $\mathcal{N} = 6.023 \times 10^{23}$ mol $^{-1}$
 Universal gas constant $R = 8.314 \times 10^7$ erg mol $^{-1}$ K $^{-1} = 8.314$ J mol $^{-1}$ K $^{-1}$
 Boltzmann's constant $k = R/\mathcal{N} = 1.381 \times 10^{-16}$ erg K $^{-1} = 1.381 \times 10^{-23}$ J K $^{-1}$

Physical properties of dry air

Molecular weight = 28.964
 Gas constant for dry air = 2.87×10^7 erg g $^{-1}$ K $^{-1} = 287$ J kg $^{-1}$ K $^{-1}$
 Specific heats: $c_p = 0.240$ cal g $^{-1}$ K $^{-1} = 1.005$ J kg $^{-1}$ K $^{-1}$ $c_v = 0.172$ cal g $^{-1}$ K $^{-1} = 0.718$ J kg $^{-1}$ K $^{-1}$, $\gamma = c_p/c_v = 1.40$
 Density of air at 0° C and 1000 mb pressure = 1.2754 g cm $^{-3}$ or kg m $^{-3}$
 (for other temperatures and pressures multiply by 0.273 p/T)

The density, viscosity, thermal conductivity of air and the diffusion coefficient of water vapour in air as functions of temperature at 1000 mb pressure

$T(^{\circ}\text{C})$	40	20	10	0	-10	-20	-40
Density ρ (g cm $^{-3}$ or kg m $^{-3}$)	1.1125	1.1884	1.2303	1.2754	1.3238	1.3761	1.4842
Dynamic viscosity μ (poise $\times 10^4$), (N.s.m $^{-2} \times 10^4$)	1.908	1.815	1.766	1.717	1.667	1.616	1.512
Thermal conductivity K (J m $^{-1}$ s $^{-1}$ K $^{-1}$) $\times 10^7$	2.71	2.55	2.48	2.40	2.32	2.23	2.07
Diffusion coefficient $D^{\dagger}$ (cm 2 s $^{-1} \times 10$), (m 2 s $^{-1} \times 10^5$)	6.47	6.09	5.92	5.73	5.54	5.33	4.94
Kinematic viscosity ν (cm 2 s $^{-1} \times 10$), (m 2 s $^{-1} \times 10^5$)	2.84	2.53	2.39	2.25	2.11	1.98	1.72
	1.72	1.53	1.44	1.35	1.26	1.17	1.01

Note: μ and K are independent of pressure, ρ , z , and D at p .

$\dagger$ There is some evidence (see Thorpe and Mason (1966)), that these values are about 10 per cent too high.

Physical properties of water substance

Molecular weight = 18.015

Gas constant for water vapour treated as ideal gas

$$= 4.615 \times 10^7 \text{ erg g}^{-1} \text{ K}^{-1} = 461.5 \text{ J kg}^{-1} \text{ K}^{-1}$$

Latent and specific heats

$T(^{\circ}\text{C})$	40	20	10	0	-10	-20	-40
L_f (cal g $^{-1}$) (J kg $^{-1}$) $\times 10^{-5}$				79.7	74.5	69.0	56.3
L_v (cal g $^{-1}$) (J kg $^{-1}$) $\times 10^{-6}$	547.7	586.0	591.7	597.3	603.0	608.9	621.7
L_s (cal g $^{-1}$) (J kg $^{-1}$) $\times 10^{-6}$	2.406	2.454	2.477	2.500	2.525	2.549	2.603
C_w (cal g $^{-1}$ K $^{-1}$) (J kg $^{-1}$ K $^{-1}$) $\times 10^{-3}$	0.998	0.999	1.001	1.007	1.020	1.040	1.140
C_i (cal g $^{-1}$ K $^{-1}$) (J kg $^{-1}$ K $^{-1}$) $\times 10^{-3}$	4.179	4.182	4.192	4.218	4.27	4.35	4.77
				0.503	0.485	0.468	0.433
				2.106	2.031	1.959	1.813

L_f , L_v , L_s are respectively the latent heats of fusion, vaporization, and sublimation.
 C_w , C_i are the specific heats of liquid water and ice respectively.

SOME USEFUL PHYSICAL CONSTANTS

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Surface tension of pure water

T ($^{\circ}\text{C}$)	30	20	10	0	-10	-20	-40
σ (dyn/cm), ($\text{N m}^{-1} \times 10^3$)	71.18	72.75	74.22	75.70	77.29	79.14	83.9†

† Extrapolated.

Density of saturated water vapour

Over water				Over ice			
T (K)	300	290	280	270	260	250	240
ρ_v (g m^{-3})	25.8	14.5	7.74	3.93	1.87	1.05	0.25

Saturation vapour pressures over pure liquid water (e_w) and over pure ice (e_i) as functions of temperature

T ($^{\circ}\text{C}$)	e_w (mb)	e_i (mb)	T	e_w	e_i	T	e_w	T	e_w
-50	0.0635	0.0393	-24	0.8826	0.6983	1	6.565	26	33.606
-49	0.0712	0.0445	-23	0.9647	0.7708	2	7.054	27	35.646
-48	0.0797	0.0502	-22	1.0536	0.8501	3	7.574	28	37.793
-47	0.0892	0.0567	-21	1.1498	0.9366	4	8.128	29	40.052
-46	0.0996	0.0639	-20	1.2538	1.032	5	8.718	30	42.427
-45	0.1111	0.0720	-19	1.3661	1.135	6	9.345	31	44.924
-44	0.1230	0.0810	-18	1.4874	1.248	7	10.012	32	47.548
-43	0.1379	0.0910	-17	1.6183	1.371	8	10.720	33	50.303
-42	0.1533	0.1021	-16	1.7594	1.505	9	11.473	34	53.197
-41	0.1704	0.1145	-15	1.9114	1.651	10	12.271	35	56.233
-40	0.1891	0.1283	-14	2.0751	1.810	11	13.118	36	59.418
-39	0.2097	0.1436	-13	2.2512	1.983	12	14.016	37	62.759
-38	0.2322	0.1606	-12	2.4405	2.171	13	14.967	38	66.260
-37	0.2570	0.1794	-11	2.6438	2.375	14	15.975	39	69.930
-36	0.2841	0.2002	-10	2.8622	2.597	15	17.042	40	73.773
-35	0.3138	0.2232	-9	3.0965	2.837	16	18.171	41	77.798
-34	0.3463	0.2487	-8	3.3478	3.097	17	19.365	42	82.011
-33	0.3817	0.2768	-7	3.6171	3.379	18	20.628	43	86.419
-32	0.4204	0.3078	-6	3.9055	3.684	19	21.962	44	91.029
-31	0.4627	0.3420	-5	4.2142	4.014	20	23.371	45	95.850
-30	0.5087	0.3797	-4	4.5444	4.371	21	24.858	46	100.89
-29	0.5588	0.4212	-3	4.8974	4.756	22	26.428	47	106.15
-28	0.6133	0.4668	-2	5.2745	5.173	23	28.083	48	111.65
-27	0.6726	0.5169	-1	5.6772	5.622	24	29.829	49	117.40
-26	0.7369	0.5719	0	6.1070	6.106	25	31.668	50	123.39
-25	0.8068	0.6322	—	—	—	—	—	—	—

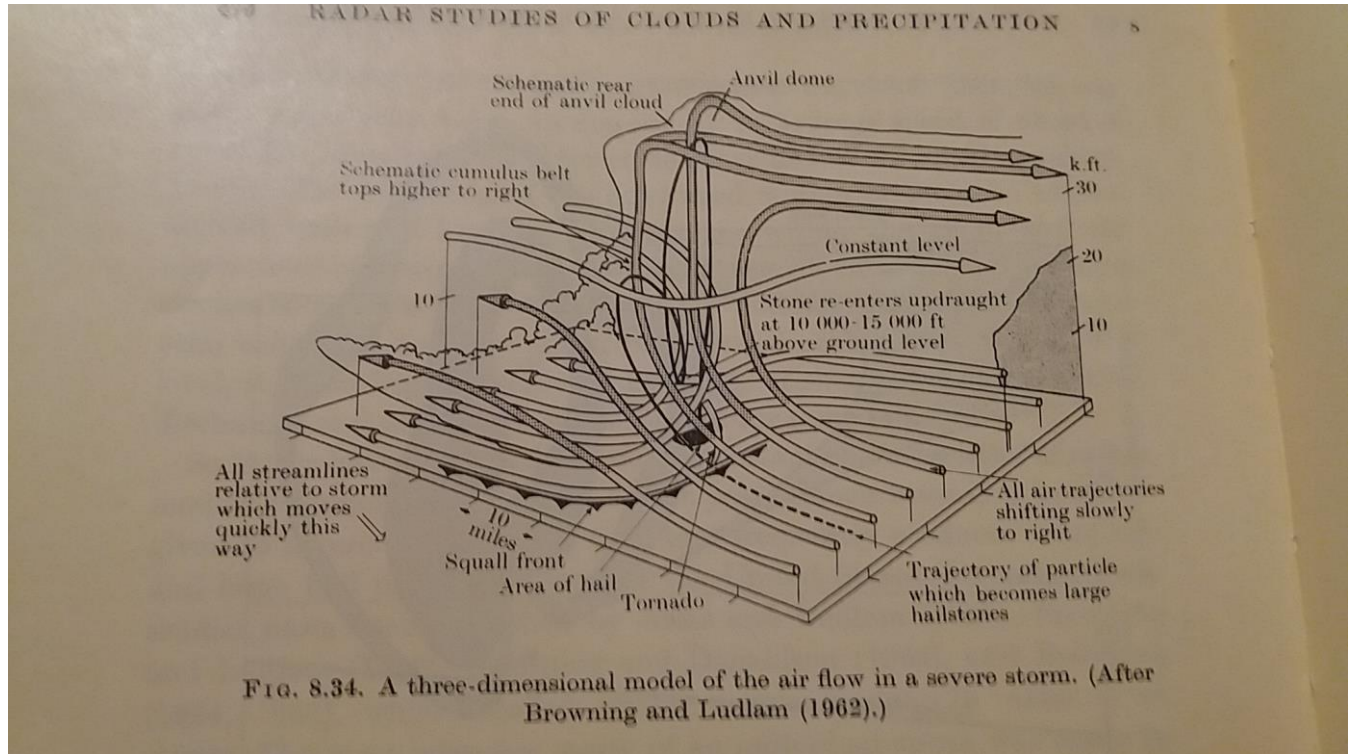
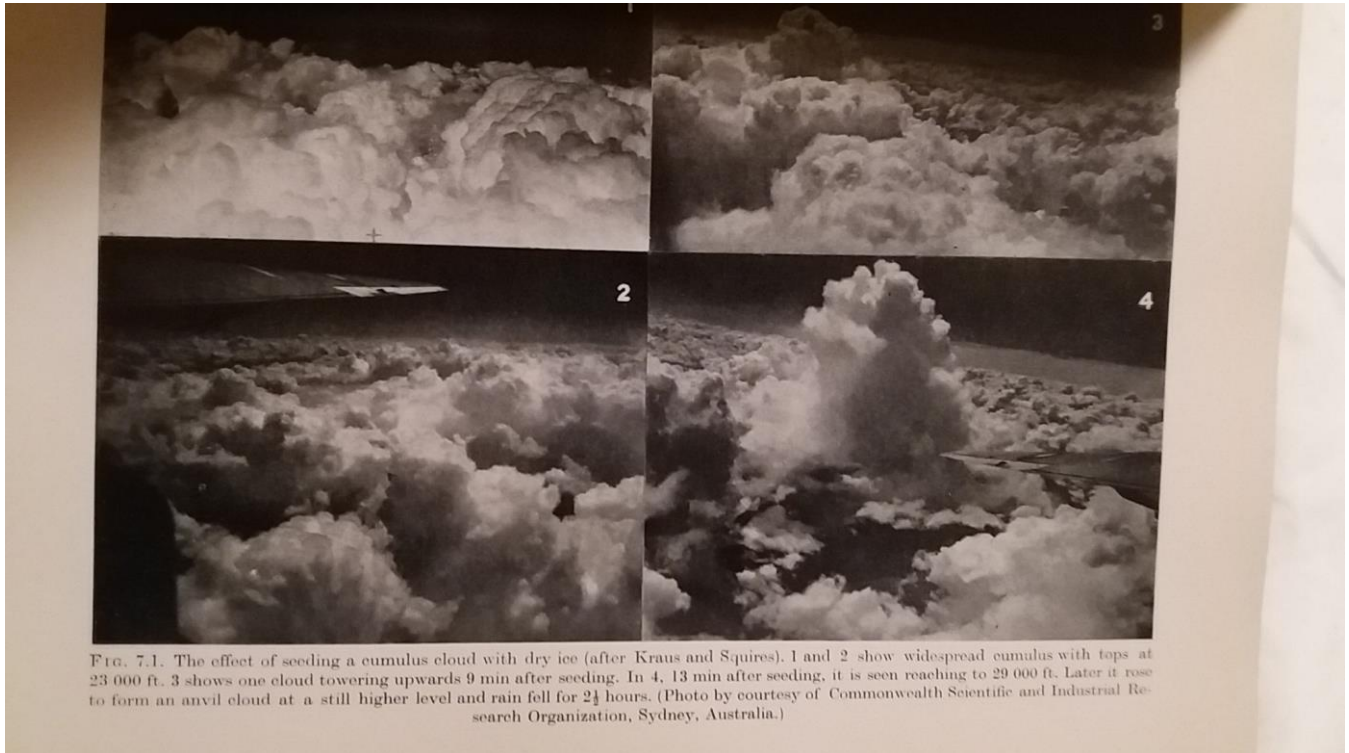




FIG. 7.2. (a) Area of New Mexico at 09.45 a.m. before cloud seeding operation. The cumulus clouds are small with bases at about 15 000 ft, their tops being supercooled. (b) Towering cumulonimbus rising from the region where a cloud was seeded 20 min earlier. (Photos copyright—Project Cirrus, Signal Corps Engineering Laboratory.)



1

The Nucleation of Water-Vapour Condensation

CLOUDS are formed by the lifting of damp air which then cools adiabatically by expansion as it encounters continuously falling pressures at higher levels in the atmosphere. The relative humidity consequently rises and eventually the air becomes saturated with water vapour. Further cooling produces a supersaturated vapour, but the excess vapour condenses on to some of the multitude of tiny particles suspended in the air to form a cloud composed of minute water droplets. The growth of these droplets tends to oppose further increase in the supersaturation, which reaches a peak value of usually less than 1 per cent and then decreases. In the absence of foreign particles and ions, much higher supersaturations are required for droplet condensation. Although we are never concerned with this process in natural clouds, we begin with a discussion of homogeneous nucleation of water vapour because it represents the simplest form of condensation process. Moreover, it provides the most straightforward illustration of the theoretical approach to nucleation problems in general, some examples of which will be discussed later.

1.1. Homogeneous condensation

It was found by C. T. R. Wilson (1897) that when air initially saturated with water vapour and freed as far as possible from foreign particles was subjected to a sufficiently large, rapid expansion, a cloud of very small water droplets appeared spontaneously from the vapour. He reported that with an initial temperature of 20°C a cloud, as distinct from a small number of larger droplets, appeared only if the expansion ratio exceeded 1.37, thus producing momentarily almost eightfold supersaturation.

In the absence of foreign particles, aggregates of the condensed phase can arise only as the result of chance collisions of molecules in the supersaturated vapour. These small aggregates (or embryos) are continually formed and disrupted because of microscopic thermal and density fluctuations in the vapour, but only if they surpass a certain critical size determined by the prevailing supersaturation and temperature, can they survive and continue to grow—otherwise they evaporate

and disappear. The supersaturated vapour is a metastable phase, and the embryos can grow to become *nuclei* for the development of the liquid phase only if they attain a size at which they are more stable than the vapour phase. The probability of formation of an aggregate of critical size increases as the supersaturation is increased. The critical radius r^* , which the aggregate, assumed spherical, must attain in order to be in (unstable) equilibrium with the vapour, was first deduced by Kelvin (1870) and later by Gibbs (1875) from thermodynamical arguments, as

$$r^* = 2M\sigma_{LV}/\rho_L RT \ln p/p_\infty, \quad (1.1)$$

where p is the pressure of the supersaturated vapour and p_∞ the equilibrium vapour pressure at temperature T over a plane surface of the liquid, σ_{LV} the specific surface energy of the liquid-vapour interface, ρ_L the density of the liquid (strictly the difference between the densities of the liquid and vapour), R the universal gas constant, and M the molecular weight of the liquid.

1.1.1.1. Thermodynamics of phase changes

Phase-equilibria and changes of phase may be conveniently discussed with the aid of two thermodynamic free-energy functions: the Helmholtz free energy

$$F = U - TS \quad (1.2)$$

and the Gibbs free energy

$$G = U - TS + pV, \quad (1.3)$$

where U is the internal energy, T the absolute temperature, S the entropy, p the pressure, and V the volume of the system under consideration. Substitution for dU from the first law of thermodynamics,

$$TdS = dU + p dV, \quad (1.4)$$

in the differential forms of (1.2) and (1.3) yields the following relations for a system of constant composition *in equilibrium*:

$$dF = -S dT - p dV \quad (1.5)$$

$$dG = -S dT + V dp. \quad (1.6)$$

Thus if a system is in equilibrium at constant temperature and volume, F is stationary, and can be shown to be a minimum. Similarly, for a system in equilibrium at constant temperature and pressure, G is a minimum. We shall be concerned mainly with systems in which the pressure is held constant and therefore with the Gibbs free energy G .

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If the system is not in equilibrium and its composition n_i (quantity of component i) is subject to change, we introduce the chemical potential

$$\mu_i = \left(\frac{\partial G}{\partial n_i} \right)_{T, p} \text{ and the change in free energy becomes}$$

$$dG = \sum_i \bar{V}_i dp - \sum_i \mathcal{S}_i dT + \sum_i \mu_i dn_i \quad (1.7)$$

We now consider a small change involving two phases, a , b , of a chemically homogeneous system, for example the condensation of an infinitesimal quantity of vapour. This involves a change $(\mu_b - \mu_a) dn$ in the chemical potential of the system and also an infinitesimal increase dA in the surface area of the liquid phase and a quantity of work $\sigma_{LV} dA$. The total change in the Gibbs free energy of the system is then

$$\Delta G = (\mathcal{S}_a - \mathcal{S}_b) dT + (V_b - V_a) dp + (\mu_b - \mu_a) + \sigma_{LV} dA, \quad (1.8)$$

where all the quantities may be taken as referring to one molecule. For an isothermal, isobaric change, $dT = 0$, $dp = 0$, and

$$\Delta G = (\mu_b - \mu_a) + \sigma_{LV} dA. \quad (1.9)$$

If the two phases are in equilibrium, then $dG = 0$, $dA = 0$, and $\mu_a = \mu_b$, so that the chemical potential of a molecule in the saturated vapour over a plane liquid surface is the same as that of a molecule in the liquid.

1.1.2. *Thermodynamic derivation of Kelvin's formula*

The thermodynamic theory of phase transitions, which rests on the assumption that the two phases are in stable equilibrium, cannot deal with the *rate* at which the transition proceeds; this is assumed to be infinitesimal, which is not strictly the case, particularly during the initial stages of development of the new phase. Nevertheless, we consider a closed system at absolute temperature T , containing vapour at pressure p , and one droplet of liquid, radius r , consisting of g molecules. Let the chemical potential per molecule in the vapour phase be μ_a and in the liquid phase be μ_b . Imagine the droplet to have been formed by the condensation of g molecules of vapour. The total change in the free energy of the system is then

$$\Delta G = (\mu_b - \mu_a)g + 4\pi r^2 \sigma_{LV} \quad (1.10)$$

$$= (\mu_b - \mu_a)g + \alpha g^{\frac{1}{3}}, \quad (1.10a)$$

where α is a constant such that $\alpha g^{\frac{1}{3}} = 4\pi r^2 \sigma_{LV}$. The second term on the right of (1.10a) is the surface free-energy term which makes an

important contribution to ΔG because of the large surface-to-volume ratio of the small droplet.[†]

The way in which the free energy ΔG varies with the size of an embryo is shown in Fig. 1.1. Curve (a) is for a saturated vapour where $\mu_a = \mu_b$, while curve (b) is for a supersaturated vapour for which $\mu_a > \mu_b$. In the latter case, there is a maximum of the free energy of the system when g has a certain critical value g^* . When the embryo has this critical size it is in equilibrium with the vapour, but the equilibrium is unstable. On surpassing the critical size, the droplet will grow with a

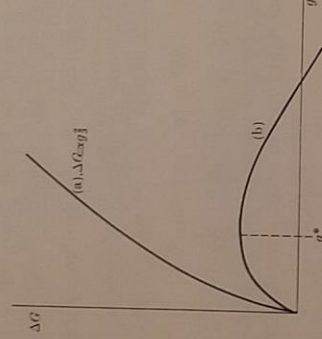


FIG. 1.1. The free energy ΔG required to form a droplet containing g molecules; curve (a) in saturated vapour; curve (b) in supersaturated vapour. g^* is the critical size.

decrease of free energy and so tend to become larger still. If, however, the vapour phase is thermodynamically stable ($\mu_a < \mu_b$), embryos of the new phase reach only a relatively small size and then decay.

The condition for equilibrium between the droplet and the vapour can be obtained by differentiating (1.10a). It is found that $d(\Delta G)/dg = 0$ when

$$\mu_b - \mu_a = -\frac{4}{3}\alpha g^{-\frac{1}{3}} = -8\pi\sigma_{LV} r^2/3g, \quad (1.11)$$

but

$$g = \frac{4}{3}\pi r^3 \frac{\mathcal{N}\rho_L}{M}, \quad (1.12)$$

[†] It is assumed that the surface tension of such a small droplet is that appropriate to a plane surface of the liquid at the same temperature. The effect of the curvature of small droplets on their surface tension has been the subject of several theoretical treatments, e.g. Tolman (1949), Kirkwood and Buff (1949), but such indirect experimental evidence as exists does not support their conclusions (see Mason 1951).

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where $\mathcal{N}$ is Avogadro's number, and therefore combining (1.11) and (1.12) we obtain for the equilibrium condition,

$$\mu_a - \mu_b = \frac{2M\sigma_{LV}}{\mathcal{N}\rho_L r^*}, \quad (1.13)$$

where r^* is the radius of the critical nucleus containing g^* molecules.

We may obtain an alternative expression for $(\mu_a - \mu_b)$ by considering the transfer of one molecule from the vapour to a plane surface of the liquid, the vapour pressure being p , the equilibrium vapour pressure p_∞ , with the temperature T remaining constant. Then, treating the vapour as an ideal gas and remembering that $\mu_a = \mu_a(p_a)$, we may write

$$\begin{aligned} \mu_a - \mu_b &= \mu_a(p) - \mu_a(p_\infty) \\ &= \int_p^{p_\infty} d\mu_a = \int_p^{p_\infty} v_a dp = kT \int_p^{p_\infty} \frac{dp}{p}, \end{aligned} \quad (1.14)$$

$$\text{or} \quad \mu_a - \mu_b = kT \ln p/p_\infty = kT \ln S, \quad (1.15)$$

where μ_a is the free energy and v_a the volume occupied per molecule at pressure p and temperature T , k is Boltzmann's constant, and $S = p/p_\infty$ is the saturation ratio.

Substituting for $\mu_a - \mu_b$ from (1.15) into (1.13) gives

$$\ln p/p_\infty = 2M\sigma_{LV}/\rho_L \mathbf{R}T r^* \quad (1.16)$$

which is Kelvin's formula (1.1).

1.1.3. Embryo droplets in statistical equilibrium with the vapour

According to (1.10) and (1.11),

$$\Delta G = \frac{1}{3} a g^3 = \frac{4}{3} \pi r^3 \sigma_{LV}, \quad (1.17)$$

and so the system is in true equilibrium (minimum value of ΔG), only when $g = 0$. For small finite values of ΔG , there is always a small probability, $e^{-\Delta G/kT}$, that the system will be found in a non-equilibrium state because of microscopic density and temperature fluctuations. If, in a system containing n_0 molecules, there are n_g embryos each consisting of g molecules, then the most probable distribution n_g is shown by Frenkel (1946) to be given by

$$n_g = n_0 \exp(-\Delta G/kT) = n_0 \exp\left(g \ln S - \frac{a g^3}{kT}\right). \quad (1.18)$$

satisfactorily confirmed by experiment over a wide temperature range. There are certainly unsatisfactory and unconvincing features of the theory, but the discrepancies between the theory and the experimental results of different authors are not systematic and cannot be reconciled by inserting different values for such parameters as the condensation coefficient and surface tension in eqn (1.36). Much of the trouble lies in the experiments where there are great difficulties in excluding entirely small traces of contamination and sources of foreign nuclei, in determining accurately the droplet concentration, the sensitive time of the chamber, and hence the nucleation rate. There remains a need for an *experimentum crucis* in which one can be certain of observing homogeneous nucleation in the absence of foreign particles and in which the droplet concentration will be sufficiently low to ensure that the peak supersaturation can be accurately computed in terms of the expansion ratio and initial temperature for a truly adiabatic system. The experiment should probably be carried out in all-glass apparatus, with a number of highly-purified carrier gases, and over a fairly wide range of temperature.

1.2. Condensation on ions

We have seen that homogeneous condensation of water vapour occurs only if the supersaturation reaches several hundred per cent, but the process is greatly facilitated in the presence of foreign particles that serve as ready-prepared aggregates (nuclei) for condensation.

In a classic investigation, Wilson (1896) discovered that, after removing foreign particles from the air of his cloud chamber by making repeated expansions and allowing the droplets to settle out, further expansions produced no droplets until the expansion ratio exceeded 1.25, corresponding to a saturation ratio S of about 4. The nuclei responsible for these droplets could not be removed by successive expansions nor by filtration through cotton wool. The fact that condensation occurred at similar supersaturations when the air was irradiated with X-rays suggested that the responsible nuclei were small ions. By ionizing the air in his chamber by a very short exposure to X-rays and by application of electric fields of the appropriate polarity, Wilson was able to produce an excess of ions of either sign. He then found that an expansion from an initial temperature of 293 K produced condensation on some negative ions with an expansion ratio of 1.25, for which $S = 4$, that practically all the negative ions were involved when the ratio exceeded 1.28, but that to effect condensation on positive ions,

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the expansion ratio had to exceed 1.31 ($S \simeq 6$). Wilson's results have been confirmed by several later workers, a representative sample of results being given in Table 1.2. The conditions under which condensation occurs upon ions seem to be more sharply defined and more reproducible than is the case for homogeneous nucleation; the results obtained by different workers are in good agreement.

TABLE 1.2
Saturation ratios at which condensation occurs on small ions

	Ion sign	T_1 (K)	T_2	V_1/V_2	S
Wilson (1899)	$\left(\begin{smallmatrix} - \\ + \end{smallmatrix} \right)$	293	267.8	1.252	4.2
Praibram (1906)	$\left(\begin{smallmatrix} - \\ + \end{smallmatrix} \right)$	293	..	1.31	0.0
	$\left(\begin{smallmatrix} - \\ + \end{smallmatrix} \right)$	293	..	1.236	3.7
	$\left(\begin{smallmatrix} - \\ + \end{smallmatrix} \right)$	293	..	1.31	0.0
Laby (1908)	-	..	267.6	1.256	4.2
Andr��n (1917)	-	..	267.8	1.253	4.1
Fowell (1928)	-	291	266.5	1.245	3.98
Flood (1933)	-	..	265	1.252	4.1
Loeb, Kip, and	$\left(\begin{smallmatrix} - \\ + \end{smallmatrix} \right)$	295	..	1.25	..
Einarsson (1938)	$\left(\begin{smallmatrix} - \\ + \end{smallmatrix} \right)$	295	..	1.31	..
Scharer (1939)	$\left(\begin{smallmatrix} - \\ + \end{smallmatrix} \right)$	292	..	1.25	4.14
Sander and	$\left(\begin{smallmatrix} - \\ + \end{smallmatrix} \right)$	292	..	1.28	4.87
Dank��ler (1943)	-	..	265	..	3.9

Note: T_1 and V_1 are respectively the initial temperature and volume of the air, while T_2 , V_2 refer to the values at the end of the expansion.

If a droplet of radius r and dielectric constant ϵ , condenses on an ion of charge q and radius r_0 in an environment of saturation ratio S and dielectric constant ϵ' , the change in the free energy of the system is

$$\Delta G = -\frac{4}{3}\pi r^3 \rho_L \frac{V}{M} kT \ln S + 4\pi r^2 \sigma_{LV} + \frac{q^2}{2} \left(\frac{1}{\epsilon'} - \frac{1}{\epsilon} \right) \left(\frac{1}{r} - \frac{1}{r_0} \right). \quad (1.39)$$

The condition for equilibrium between the droplet and the vapour is given by $\partial \Delta G / \partial r = 0$ which, if we assume $\partial \sigma_{LV} / \partial r = 0$ and $\epsilon' = 1$, gives

$$\frac{R T \rho_L}{M} \ln S = \frac{2 \sigma_{LV}}{r} - \frac{q^2}{8 \pi r^4} \left(1 - \frac{1}{\epsilon} \right). \quad (1.40)$$

Comparing this with the Kelvin equation (1.1), it is seen that the saturation ratio S at equilibrium is smaller for a charged than an uncharged droplet. Eqn (1.40), expressing the variation of the equilibrium supersaturation with the droplet radius, is represented by the

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full curve of Fig. 1.5. The value of $r = r_1$ at which β attains a maximum value is given by $r_1^3 = q^3(1 - 1/\epsilon)/(4\pi\sigma_1\gamma)$; when the droplet exceeds a radius r_1 , it will continue to grow with a decrease of free energy as long as the supersaturation is maintained, and the nucleus droplet becomes a stable aggregate of the condensed phase.

In order to calculate the rate at which droplets will, under the influence of statistical fluctuations, grow from an initial radius $r_0 (< r_1)$ to a radius $r_0 (> r_1)$ that will be in equilibrium with the prevailing saturation ratio β , Tohmfor and Volmer (1938) modified the

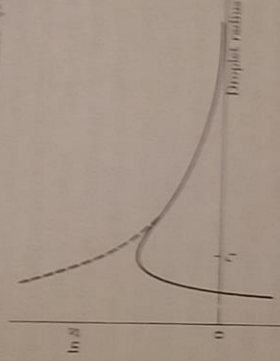


FIG. 1.5. The equilibrium vapour pressure for small droplets (full curve; charged droplets; broken line; uncharged droplets).

Becker-Döring theory and used eqn (1.39) to obtain the change in free energy ΔG^* involved in the formation of a droplet of critical size. They derived the following expression for the rate of production of droplets at 265 K;

$$I \simeq 10^7 N_1 \exp(-\Delta G^*/kT), \quad (1.41)$$

where N_1 is the number of ions per cm^3 in the vapour. On substituting experimental values of $S = 4.1$, $T = 265$ K, $\sigma_1/\gamma = 77 \text{ erg cm}^{-2}$, $q = 4.8 \times 10^{-10} \text{ e.s.u.}$, $I = 1 \text{ cm}^{-3} \text{ s}^{-1}$, and $N_1 = 10^3 \text{ cm}^{-3}$ in (1.39), (1.40), and (1.41), Tohmfor and Volmer found that the equations could be satisfied simultaneously only if $r_0 = 4.8 \times 10^{-8} \text{ cm}$, $r_1 = 10.2 \times 10^{-8} \text{ cm}$ and $\epsilon = 1.85$. This is not an entirely convincing result because, although the effective dielectric constant ϵ in the strong field of an ion is likely to be considerably less than the static value of 80 for bulk water, it is unlikely to be less than the high-frequency value of 3. This apart, it is clear that the above theory is incomplete in that it fails to account for the fact that negative ions promote condensation at lower supersaturations than do positive ions.

This simple treatment ignores polarization of the droplet by the ion. In water, where the strongly polar molecules form an oriented surface layer, the surface energy of this droplet will probably be modified to an extent depending not only on the magnitude of the charge as indicated by eqn (1.39), but also on its polarity. Therein may lie the explanation of negative ions being able to promote condensation at lower supersaturations than can positive ions.

Loeb, Kip, and Edinsson (1938), who obtained experimental confirmation of this sign preference, argue that small ions cannot grow into droplets by successive attachments of water vapour molecules, but rather that an ion is captured by a molecular aggregate or embryo, which is not yet large enough to possess a regular surface layer and a statistically conditioned electrical double layer that are characteristic of a true droplet. The molecules in the surface layer of the embryo are imagined to be oriented with the oxygen atoms directed outwards; the surface forces field will then tend to orient approaching vapour molecules with their protons towards the surface, so favouring H-bond linkages and propagation of the surface structure. The capture of a negative ion by the aggregate will enhance the surface forces field and so favour capture of vapour molecules in the correct orientation, while molecules striking a positively charged embryo would have to rearrange themselves before becoming bound. Thus, condensation may be expected to occur more readily on negative ions than on positive ions.

Again, it appears that eqn (1.46), which is derived in terms of macroscopic concepts, cannot apply to the pseudo-crystalline embryo droplets just discussed; indeed, the existence of sign preference, which is not predicted by (1.46), is evidence of the breakdown of this equation in the very early stages of the condensation process.

1.3. Nucleation by insoluble particles

The atmosphere is never sufficiently clean for condensation to occur homogeneously, or upon ions. Particles of dust and smoke, hygroscopic crystals, and small droplets of solution may also serve as centres of condensation. We shall now discuss the properties of these condensation nuclei beginning with insoluble particles.

Volmer (1939) extended the original Volmer-Weber nucleation theory to treat the case of condensation on a plane, solid surface. A condensing droplet is assumed to take the shape of a spherical cap characterized by its radius of curvature r , and ϕ , the angle of contact of the liquid on the substrate—see Fig. 1.6. Using the subscripts L, C, and V to

1.1.3 NUCLEATION OF WATER VAPOR CONDENSATION. In dense the liquid, nucleating substance, and vapour, Fletcher (1961) writes

$$m = m_0 \phi = (n_{L0} - n_{V0})/n_{V0} \quad (1.43)$$

and the change in free energy involved in forming the droplet is

$$\Delta G = n_L \bar{M} (n_0 - n_0) V_L + n_{V0} A_{LV} + (n_{L0} - n_{V0}) A_{VL} \quad (1.44)$$

where

$$A_{LV} = 2\pi r^2(1 - \cos \phi), \quad A_{VL} = \pi r^2(1 - \cos \phi), \quad V_L = \frac{\pi r^3}{3} (3 + \cos \phi)(1 - \cos \phi).$$

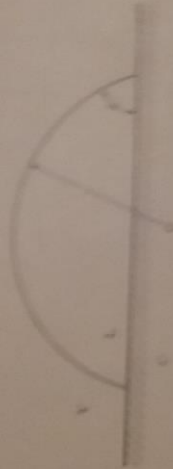


FIG. 1.6. A spherical cap embryo of liquid (L) in contact with its vapour (V) and a nucleating surface (S).

The radius of the critical embryo is obtained by putting $\partial/\partial r(\Delta G) = 0$ to give

$$r^* = 2M\sigma_{LV}/\rho_L R T \ln(p/p_\infty) \quad (1.45)$$

and, after substitution in (1.43),

$$\Delta G^* = \frac{16\pi M^2 \sigma_{LV}^2}{3(\rho_L R T \ln(p/p_\infty))^3} f(m), \quad (1.46)$$

where

$$f(m) = (2 + m)(1 - m)^{3/4}, \quad (1.47)$$

These results differ from those for homogeneous nucleation only by the factor $f(m)$ in the expression for ΔG^* . Since $-1 \leq m \leq 1$, $0 < f(m) < 1$, so that the presence of a foreign surface reduces the free energy necessary to form a critical embryo except in the special case of the surface being completely hydrophobic ($\phi = 180^\circ$).

Calculation of the nucleation rate follows the same lines as for homogeneous nucleation and yields the following expression for the rate of production of critical embryos on unit surface area:

$$I \approx (p/(2\pi M R T))^4 \sigma r^{*2} \exp(-\Delta G^*/kT), \quad (1.48)$$

in which ΔG^* is given by (1.46), $p/(2\pi M R T)^4$ is the rate of impact of molecules on unit area of surface, σr^{*2} is the surface area of the critical

produced mixed droplets of size 10^{-4} cm, shown in Fig. 1.10(c), may be compared with those for a pure solution droplet in Fig. 1.10(a).

1.6. Concluding remarks

It appears, then, that atmospheric aerosols consist of solid insoluble particles, of soluble matter giving rise to pure solution droplets, and of mixed nuclei that are partly soluble, partly insoluble. The last two groups of nuclei, being hygroscopic, will obviously be preferred as centres of condensation, their efficiency being determined by both their size and chemical nature. Whether or not non-hygroscopic particles will play a significant role in cloud formation will depend largely on the numbers and sizes of the more favoured hygroscopic nuclei present. In any case, because insoluble particles may obtain a hygroscopic film, either by capturing small solution droplets or by acting as condensation platforms for trace impurities, it may be that truly non-hygroscopic particles are rather rare in the atmosphere.

a production rate of $4000 \text{ cm}^3 \text{ s}^{-1}$ in areas of breaking waves as reasonable in the light of some laboratory experiments that simulated the violent agitation of the sea surface, but there is a need for much better designed experiments capable of determining the total production rate and size spectrum of salt nuclei under realistic oceanic conditions. Meanwhile, an average production rate of $100 \text{ cm}^3 \text{ s}^{-1}$ for the oceans as a whole, represents the best estimate that can be made at present. This may be compared with Squires' (1966) estimate of $500 \text{ cm}^3 \text{ s}^{-1}$ for the average production rate of potential cloud-forming nuclei over the land mass of the Northern Hemisphere, and suggests that while sea-salt particles are probably a major source of condensation nuclei in maritime clouds, they make only a minor contribution to clouds forming well inland.

2.4.3. *Production of nuclei over the continents*

Of the natural processes involved in the primary production of aerosol particles over land, the more important are: combustion (e.g. forest fires, volcanoes); chemical reactions involving trace gases particularly in the presence of water, either in cloud and fog droplets or on the surfaces of solid particles; the raising of soil and dust particles from the earth's surface; the emission of pollen and volatile substances by plants. In addition, man-made activities, notably combustion, contribute large quantities of nucleogenic matter in both vapour and particulate form. Despite the increasing scale of atmospheric pollution in modern times, one would expect natural sources of cloud-forming nuclei to outweigh man-made sources unless the constitution of clouds has changed markedly over the last two hundred years. Squires (1966), from measurements made in SE. Australia and Colorado, U.S.A. of the concentrations of nuclei that become active at supersaturations of 0.5 per cent in a thermal diffusion chamber, and from estimates of the residence times of the air over these semi-arid regions, deduced the average production rate of such nuclei at the surface to be $500 \text{ cm}^3 \text{ s}^{-1}$. Allowing for the fact that land covers only 40 per cent of the total area, he arrived at an overall rate of $200 \text{ cm}^3 \text{ s}^{-1}$ for the production of cloud-forming nuclei of natural terrestrial origin over the Northern Hemisphere. Squires also attempted an assessment of the production of man-made nuclei over Denver, Colorado, by determining the transport of particles downwind of the city. Measurements made on four different days, on which the wind distribution with height was similar, indicated the average production rate of Aitken nuclei over the city to be

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$10^5 \text{ cm}^{-2} \text{ s}^{-1}$, and that for larger nuclei active at 0.5 per cent supersaturation to be $10^4 \text{ cm}^{-2} \text{ s}^{-1}$. On the assumption that man-made production of such nuclei is proportional to the total consumption of fuel of all kinds within any region, the results for Denver were extrapolated to larger regions—Colorado, the United States, and the Northern Hemisphere—for which the production rates of the larger 'cloud' nuclei were estimated to be about 20 , 70 , and $10 \text{ cm}^{-2} \text{ s}^{-1}$ respectively. In other words, the man-made contribution was estimated to be 14 per cent of the total for the United States and only 5 per cent for the Northern Hemisphere as a whole. Since the oceans probably make only a minor contribution to the total supply of cloud-forming nuclei, the implication is that the major sources are natural combustion processes and chemical reactions involving the absorption of such vapours as SO_2 , H_2S , NH_3 , and Cl by cloud droplets or on the surfaces of wetted solid particles.

There is considerable geological and other evidence that mineral dusts are transported and deposited over distances of hundreds, or even thousands, of miles from their source. Unfortunately, investigations have been limited to particles larger than $1 \mu\text{m}$ and almost nothing is known about the quantities of sub-micron dust particles in the atmosphere. In dust storms, the mass distribution of the particles near the ground has a maximum at about $25\text{-}\mu\text{m}$ radius but this value decreases with increasing height and increasing distance from the source. Quartz particles deposited in the Pacific Ocean are mostly between 0.5 and $15 \mu\text{m}$ in radius, the distribution having a maximum at a settling radius of $2\text{--}5 \mu\text{m}$. Twomey (1960) provides evidence that desiccation and heating of soil can generate $10^4\text{--}10^5$ particles/ cm^2 of surface. This is not a very efficient source unless the process is repeated many times under changing meteorological conditions. He thinks that the particles, which seem to be as small as $0.01 \mu\text{m}$, are formed by crystallization of salt dissolved in the soil water or by thermally induced rupture of salt layers on the soil surface.

2.4.4. The growth of nuclei by coagulation

In a dense population of nuclei, chance collisions lead to coagulation of particles, an increase in their size, and a corresponding decrease in their concentration. It is an experimental fact (see Whytlaw-Gray and Patterson (1932)) that, if in a given volume of smoke the number of particles is plotted against the time, a linear relation of the form

$$\frac{1}{n} = Kt \text{ is obtained, where } n \text{ is the concentration after time } t, n_0$$

2.4.2. *Production of sea-salt nuclei*

It was once thought that sea spray, produced in the foam of breaking waves, was the main source of sea-salt nuclei. However, apart from those that originate in surf, such particles are not very numerous and are generally too large to remain airborne for very long. In recent years Woodcock and his co-workers (Woodcock, Kientzler, Arons, and Blanchard 1953, Kientzler, Arons, Blanchard, and Woodcock 1954, and Blanchard and Woodcock 1957), following earlier suggestions by Jacobs (1937), Owens (1940), and Boyce (1954), have demonstrated by high-speed photography that salt particles larger than $1\text{ }\mu\text{m}$ are produced mainly by the bursting of numerous small air bubbles in the foam of breaking waves. The photographs of Fig. 2.15, taken by Knelman, Dombrowski, and Newitt (1954), show that the spherical bubble cap bursts near the summit, where it is thinnest, and the air, in escaping through the hole, ruptures the film. After bursting of the cap, water rushes into the crater to form an unstable jet, and this breaks up to produce between one and five drops that are about one-tenth the diameter of the parent bubble. Bubbles of about 2-mm-diameter project to heights of nearly 20 cm but, for both larger and smaller bubbles, the maximum height of projection is less than this. Blanchard and Woodcock (1957) suggest that the lower size limit of the

bursting bubbles is determined by their dissolution in sea water. The smallest bubbles observed to reach the ocean surface in 'white cap' were about $100\text{ }\mu\text{m}$ in diameter and those producing salt particles $m \sim 10^{-11}\text{ g}$. Much smaller bubbles with a mean diameter of $40\text{ }\mu\text{m}$ were observed only when snowflakes melted in sea water. Under these conditions, the lower size limit of the bubbles is not small enough to explain the production of the smaller salt nuclei by the evaporation of drops from the breaking bubble jets.

But, in addition to these drops, Mason (1964) found that bubbles larger than about 2 mm in diameter produced a few drops with diameters between 5 and $30\text{ }\mu\text{m}$ and containing between 2×10^{-14} and $5 \times 10^{-10}\text{ g}$ of salt; these were found very close to the water surface and were apparently ejected at low angles to the horizontal. Also, by studying the bursting of bubbles in a cloud chamber, Mason discovered that the disruption of the bubble cap produced much larger numbers of particles too small to be detected by conventional methods. Clean filtered air was passed through identical jets into two chambers, one containing sea water, and the other, distilled water. After an equal number of bubbles had burst in each chamber, an expansion was made; a dense cloud of tiny droplets was then observed in the space above the bubbles but not above the distilled water. It appeared that the bursting bubbles produced very small salt particles that acted as condensation nuclei during the expansion. It was estimated that bubbles of between 0.5 and 3 mm in diameter produced $100\text{--}200$ nuclei with salt contents between 10^{-10} and 10^{-14} g . In a later paper, Mason (1967) revised this figure to 300 ± 80 nuclei per bubble almost independent of bubble diameter in the range $0.25\text{--}2.15\text{ mm}$. This result was confirmed by Twomey (1960) using a diffusion cloud chamber, but Blanchard (1963), using both a Scholze counter and a diffusion cloud chamber, found that the number of bubble-film droplets produced by *single* bubbles decreased with decreasing bubble diameter below 2 mm and that bubbles of $d < 200\text{ }\mu\text{m}$ produced no droplets.

This question has been carefully re-examined by Day (1966), working in Mason's laboratory and photographing the bursting of individual bubbles in the stable supersaturated atmosphere of a diffusion cloud chamber. From photographs of the kind shown in Fig. 2.16, Day confirmed that the average number of bubble-film droplets is dependent on bubble size, and ranges from none for bubbles of $d < 100\text{ }\mu\text{m}$, to $300\text{--}400$ for bubbles of $d = 4\text{ mm}$ bursting in sea water and saline solutions. His results are plotted in Fig. 2.17, and those of Mason

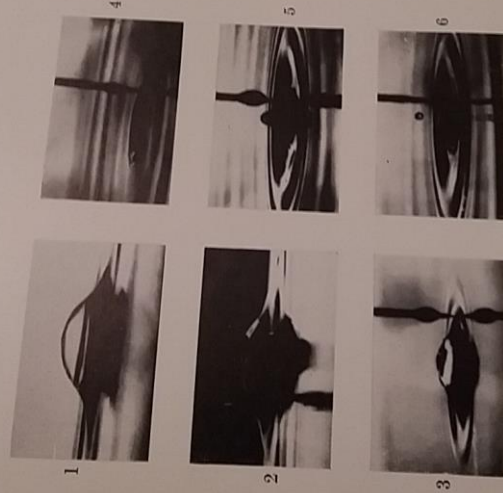


FIG. 2.15. Six stages in the collapse of an air bubble of 3 mm diameter in water. In the initial stage a protuberance develops on the hemispherical surface of the bubble (2), the collapse of which produces a cloud of fine droplets and leaves a large gap in the bubble envelope (3). The escape of gas through this gap disperses the cloud and imparts to the droplets a high velocity. Thereafter a crater is formed at the site of the bubble (4), and the incoming rush of liquid produces an unstable jet (5), which breaks up into one or more large drops (6). (From Knelman *et al.* (1954).)

surface, either as droplets of sea-water over the oceans, or as soil and mineral-dust particles over the continents; (4) by coagulation of nuclei leading to the formation of larger particles of mixed constitution.

2.4.1. Production of nuclei by combustion and by chemical reactions
In combustion processes the substances are molecularly dispersed; supersaturated and either droplets or solid particles are strongly according to the nature of the particular substance. Typical substances formed in large quantities in this way are ashes, soot, tar products, and oils, as well as sulphuric acid and sulphates in cases where the fuel contains sulphur. A great variety of particles is formed in this way by industrial operations, by natural and man-made fires. The sizes of the particles cover a wide range, the great majority lying within that of the Aitken nuclei.

Some data on average concentrations of sulphur, ammonia, and nitrogen compounds, and sea salts found in the air, in aerosols, and in rain-water are given in Table 2.4 together with their likely sources and sinks. In many cases the information is based on the results of measurements made at only a few places and must therefore be regarded as tentative and not necessarily representative of the whole atmosphere.

Sulphur. In the gaseous form sulphur exists mainly as SO_2 and H_2S . Most of the natural sulphur is believed to exist as H_2S , produced by bacterial decomposition of organic matter and reduction of sulphate in the soil and in sea-water, but there have been very few measurements of the H_2S (as opposed to the total sulphur) content, especially in unpolluted atmospheres. Junge (1960) has made some measurements near Boston and found the concentration of H_2S to be fairly constant at about $9 \mu\text{g}/\text{m}^3$ with the SO_2 content more variable but, on the average, about 2.5 times larger. Jacobs, Braverman, and Hooheiser (1957) found the mean concentration of H_2S in New York City to be $3 \mu\text{g}/\text{m}^3$ and the concentration of SO_2 to vary between 280 and 1500 $\mu\text{g}/\text{m}^3$. Since H_2S cannot be removed rapidly in the gaseous phase by solution in cloud and rain drops, and because the estimated average residence time of local sulphur in the atmosphere is only about 20 days, it seems likely that H_2S must be partially converted to sulphate, probably in cloud droplets or on the surfaces of particles. Oxidation of H_2S to SO_4 is indicated by the fact that the SO_4 content of rain in rural areas is higher in the summer, whereas in polluted regions it is higher in the winter because of the higher SO_2 production.

of large particles (or droplets), which are more easily captured, and of prominent crystals such as NaCl, which are easily identified.

2.4. The production of natural aerosols

The main sources of raw material for the manufacture of atmospheric aerosols are: the gases released into the atmosphere from land and ocean surfaces, and by plants; trace gases produced by reactions involving the atmospheric constituents including water vapour; particles of soil, mineral dust, and sea salts carried up from the earth's surface by the wind; and a wide variety of particulate and gaseous products of combustion. The materials so produced are present in small concentrations of order 1 to 100 $\mu\text{g}/\text{m}^3$, which usually show large variations in space and time because they spend only a relatively short time in the atmosphere before being removed by rain. Chemical analyses of the large and giant nuclei, and of rain water, show the major constituents to be sulphate, chloride, nitrate, and ammonia. The fact that sulphur, ammonia, and nitrogen oxides are usually present in much higher concentrations in the gaseous than in the particulate phase suggests that these gases, which have been identified in the form of SO_2 and H_2S , NH_3 , N_2O , and NO_2 , are partially converted into particulate matter and are then attached to clouds and raindrops. Atmospheric aerosol is believed to originate in four main ways: (1) by condensation and sublimation of vapours during the formation of smokes by heat and combustion; (2) by chemical reactions between trace gases produced either from natural sources or by combustion, and involving, perhaps, water vapour, ultraviolet radiation, and solid particles; (3) by the mechanical disintegration and dispersal of matter at the earth's

particles cover a wide range of sizes, from the smallest Aitken nuclei.

Some data on average concentrations of nitrogen compounds, and sea salt in rain-water are given in Table 2.2. In many cases the information is based on measurements made at only a few places, and is therefore tentative and not necessarily representative.

Sulphur. In the gaseous form, sulphur is present in the atmosphere. Most of the natural sulphur is produced by bacterial decomposition of organic matter in the soil and in sea-water, but some is also produced by the oxidation of the H_2S (as opposed to SO_2) in unpolluted atmospheres. Dr. J. H. Seinfeld near Boston and found the concentration of H_2S at about 9 $\mu\text{g}/\text{m}^3$ with the average, about 2.5 times as much. Dr. J. H. Seinfeld (1957) found the mean concentration of H_2S to be 3 $\mu\text{g}/\text{m}^3$ and the concentration of SO_2 to be 1 $\mu\text{g}/\text{m}^3$. Since H_2S cannot be dissolved in cloud and rain, the residence time of local H_2S is it seems likely that H_2S is probably in cloud droplets. The concentration of H_2S to SO_4 is indicated to be higher in the winter because of

temperatures measured in our experiments leaves little doubt that the latter were concerned with the homogeneous nucleation of water drops and provides supporting evidence that the 'flickering cluster' model of liquid water has some validity. A more detailed discussion may be found in Fletcher (1970).

4.3. Evidence for the supercooling of water below -40°C

Although it has been established, by several different workers, that micron-size droplets of very pure water freeze within less than a second

after being supercooled to -41°C , and that this temperature is generally accepted as the lower limit for their spontaneous nucleation; it should be mentioned that evidence has been offered to suggest that water may exist in the liquid state at considerably lower temperatures.

One such claim was made by Rau (1944), who stated that when all foreign nuclei were removed (or rendered impotent), it was possible to them at that temperature for several hours. Below -72°C , the drops froze to produce crystals in the form of either octahedra, hexahedra or cubes in contradistinction to the hexagonal crystals formed at higher temperatures. The crystals melted on being warmed above -72°C . Rau suggested that at -72°C spontaneous freezing occurred without the aid of nuclei, the result being a new crystalline ice structure. Attempts to repeat these observations have been made by Cooley (1947b) and Brewer and Palmer (1951), who reported that they could not reproduce Rau's results unless they deliberately contaminated their water drops with fairly high concentrations of organic vapours such as acetone or alcohol, and criticized Rau's experimental arrangements on the grounds that alcohol contamination could have arisen from his cooling system. In a private communication, Rau says that he has repeated his experiments, eliminating any possibility of contamination of his water drops by alcohol, and that he has been able to reproduce his earlier results. Unfortunately, he gives so little information on his experimental arrangement, the methods he uses to remove impurities and to determine the physical state of his condensate (visual inspection is not good enough), that it is very difficult to assess his work, which will require independent confirmation before it is accepted. The fact that his crystals melted at -72°C makes it very difficult to believe that they could have been pure ice in any form. Evidence suggesting a phase change at about -62°C was obtained by Sander and Duck (1947) (1943), who measured the critical supersaturation for the precipitation from water vapour in air cooled, by rapid expansion in a cloud chamber down to temperatures of -75°C . Until a terminal temperature of -62°C was reached, the precipitate appeared to consist of spherical particles that were reported as droplets. Below -62°C , the precipitate appeared to consist of angular ice crystals. These authors suggested that either the crystals were formed by direct nucleation of ice from a vapour, or that -62°C represented the crystallization temperature what, at higher temperatures, had been liquid droplets.

This work has been repeated rather more carefully by Mackenzie et

(1961) using clean nitrogen as the carrier gas. Their results are very similar to those of Sander and Damköhler, the appearance of spherical particles rapidly giving way to a cloud of glittering ice crystals at temperatures below -65°C . Madonna *et al.* suggest, though, that the non-glittering particles appearing at temperatures between -41°C and -65°C need not necessarily be liquid droplets but frozen spheres which have not developed crystalline faces. A similar suggestion was made earlier by Mason (1952*b*) to account for the appearance of iridescence in mother-of-pearl clouds that occur in the stratosphere at heights of about 23 km and temperatures of -80°C . No evidence for the sudden appearance of ice crystals below -65°C was obtained by Maybank and Mason (1959) who determined the numbers of ice crystals produced by the explosive expansion of a small volume of moist air for various values of the terminal temperature and supersaturation. In clean air, crystals appeared only when the supersaturation surpassed 400 per cent and the terminal temperature fell below -40°C , which suggests that they formed by condensation upon ions followed by homogeneous freezing. With increasing expansions, producing larger supersaturations and lower temperatures, the concentration of crystals increased continuously up to a maximum value of $4 \times 10^6 \text{ cm}^{-3}$. There was no discontinuity in the curve at -65°C or any other temperature, and there was no evidence for the crystals being produced by direct sublimation of the vapour rather than by spontaneous condensation followed by instant freezing, even if the distinction were a profitable one under these conditions.

In conclusion then, there appears to be no convincing evidence for believing that small droplets of pure water can be supercooled below about -41°C for longer than about 1 s.

4.4. Ice nuclei in the atmosphere

4.4.1. *Experimental techniques*

The most commonly used methods of measuring the concentration of ice nuclei in the atmosphere involve cooling the air sample to form a supercooled cloud at a known temperature, and counting the ice crystals that form either by the freezing of droplets containing ice nuclei or by direct condensation on the nuclei themselves. A supercooled cloud may be formed by chilling an air sample below its dew point. The cooling may be achieved either by adiabatic expansion or by direct cooling within a refrigerated chamber. Ideally, the a

TABLE 5.1. *Classification*

Type of particle (P)	TERM			
	CODE SYMBOL	TYPICAL FORMS	Plates	Stellar crystals
	1			
	2			
	3			
	4			
	5			
	6			
	7			
	8			
	9			
	0			
Additional characteristics	m			
	r			
	f			
	w			
	$(*)$			
Size of particle (D)	a	0-0.49 mm		
	b	0.5-0.99 mm		
	c	1.0-1.99 mm		
	d	2.0-3.99 mm		
	e	4.0 mm or larger		

of Solid Precipitation

833.333.3

also combinations of plates with or without very fine intersecting striae

also parallel striae with very short intersecting columns

and combinations of columns

and combinations of needles

spatial combinations of four or more plates

columns with plates on either (or both) side

irregular aggregates of microscopic crystals

isometric shape, central crystal cannot be recognized

ice shell, inside mostly wet

The size of the particle means the greatest extension of a particle for growth in many are considered. For a cluster of crystals it refers to the average size of the crystals composing the flake

showing a progressive increase in the fall speeds of larger drops at lower levels.

8.6.2. *Measurements of air motions and precipitation growth in showers and thunderstorms*

We have seen that in widespread 'steady' rain from stratiform clouds, where the vertical air motions are small compared with the falling speeds of the particles, pulsed Doppler radar can determine the rain-drop size spectra and their variation with height. Conversely, if limits can be set to the terminal velocities of the particles, the Doppler velocities can be corrected to give bounds for the magnitudes of the vertical motions. For example, if the radar echo from a cloud shows a well-marked melting band, this indicates the presence of snowflakes above the 0°C level, with terminal velocities of about 1 m s^{-1} ; in such a case the absence in the Doppler spectrum of velocities greater than 1 m s^{-1} would indicate the vertical motions to be weak. In the presence of steady updraughts or downdraughts, the Doppler spectrum would be displaced; knowledge of the largest or smallest falling speeds of the contributing particles in the sampling volume would then enable the air motions to be deduced.

This technique was applied successfully by Probert-Jones and Harper (1961), who noticed that, in modest shower clouds, the particles consistently accelerated in falling through the melting level, the particles above that level. They therefore corrected the largest recorded Doppler velocities by 1 m s^{-1} to give the vertical air motions at all levels above the 0°C level. In extending the analysis below the melting level, continuity of vertical motion across the melting zone was assumed in order to calculate the terminal velocities of the drops just below it.

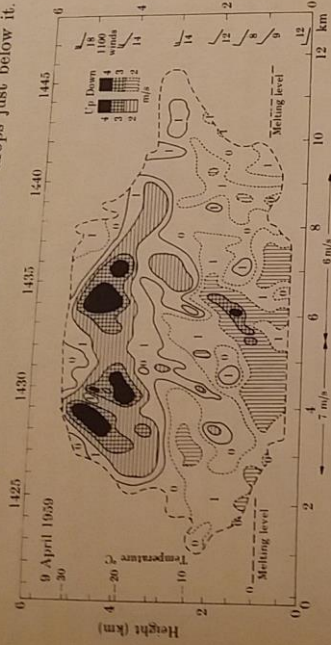


FIG. 8.25. The pattern of vertical motion in a shower cloud. The left-hand side is the leading edge of the shower. (From Probert-Jones and Harper (1961).)

The vertical air motion was then deduced from the maximum downward velocities of the Doppler spectrum on the assumption that the *largest* drops did not accelerate appreciably during their fall from just below the melting zone to the ground.

Fig. 8.25 shows a height-time analysis of the vertical motions so derived for a shower that produced 0.7 mm of rain at the ground, with a maximum rate of 8 mm h^{-1} for a short period near the start. The terminal velocities of the largest drops, as inferred from the Doppler record, were 7 m s^{-1} falling later to 6 m s^{-1} , the corresponding diameters being 2.5 and 2.0 mm . Fig. 8.25 shows the upward air motion to be concentrated in the top forward half of the cloud, while the downward motion is mainly below. The strongest upward motion of 4 m s^{-1} is close to the top of the echo, which reached 5.4 km (temperature -31°C). The downward motion is shown as mainly 2 m s^{-1} , but there is one cell of $3\text{--}4 \text{ m s}^{-1}$ in the middle of the shower, just above the

melting level. The initial echo appeared well above the 0°C level, between 2 and $2\frac{1}{2}$ km, and it was 5 min before the shower reached the ground. During the decay stage of the shower, beyond 1440 h, the echo above the melting level was almost entirely associated with downward velocities of $1\text{--}2\text{ m s}^{-1}$, and therefore almost certainly due to the fall out of snow. The pattern of vertical motion, though quite complex,

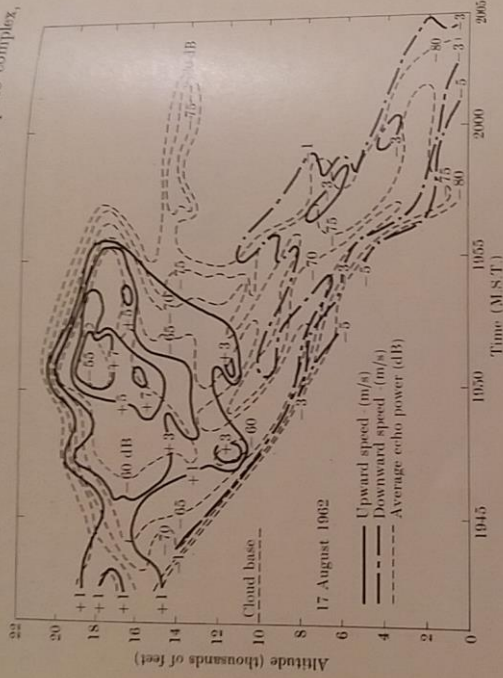


FIG. 8.26. Height-time contour map of range-normalized signal intensity, updraught speeds, and downdraught speeds, in a cumulonimbus passing over a vertically pointing Doppler radar. (After Battan (1964).)

was not complicated on this occasion by variation of wind with height and shear of precipitation across the plane of the height-time section. A detailed study of seven additional showers is described by Caton (1967).

A rather different technique has been used to investigate thunderstorms in Arizona by Battan and his colleagues. The results of Battan (1964) are shown in Fig. 8.26, where he has superimposed isopleths of updraughts and downdraughts on range-normalized contours of echo intensity. Prominent features of the diagram are the coincidence of the updraught and reflectivity maxima towards the top of the storm echo, the general similarity of the updraught and reflectivity patterns above cloud base, and the development of a moderately strong downdraught

in the rain streamer below cloud base, especially near its leading edge where reflectivity decreases sharply downward. The decrease in reflectivity indicates strong evaporation of the hydrometeors in the dry Arizona environment, while the downdraught itself may be attributed to the evaporative cooling. When the first radar echo appeared, the cloud containing only weak updraughts, but these attained a maximum speed of 7 m s^{-1} and, within 3 min, a downdraught appeared in the lower part of the cloud. The updraught and downdraught intensified together, and the former is seen to consist of several maxima appearing at successively greater heights, suggesting a series of rising thermals. In the later stages of the storm, at about 1958 hours, a melting band appeared at about 14 000 ft (4.2 km). Having inferred the vertical velocities of the air, Battan took the difference between these and the maximum downward Doppler velocities to give the terminal velocities of the largest hydrometeors. He estimated that during the early life of the echo, the largest drops were 1–2 mm in diameter but, later on, particles with terminal velocities of 15 m s^{-1} appeared at about 11 000 ft and were therefore likely to be hail pellets, ranging from 5 to 9 mm in diameter, depending upon their density. Lightning and thunder occurred at 1955 hours, 10 min after hydrometeors had started to fall towards the ground.

A similar analysis of a growing thunderstorm by Battan and Theiss (1966) revealed many similar features, including the presence of hail pellets about 1 cm in diameter. Battan, Theiss, and Kassander (1964) made observations on a decaying thunderstorm and noted that, although the updraughts up to the altitude of 28 000 ft did not exceed 0.5 m s^{-1} , the cloud nevertheless continued to produce lightning strokes at 10-min intervals. There were, however, extensive regions between 15 000 and 25 000 ft with downward Doppler velocities of 5 m s^{-1} that indicated the presence of hail pellets.

Atlas (1964) has criticized Battan's method of computing the downdraughts below cloud base in Fig. 8.26. Battan assumed the presence of detectable quantities of 0.3-mm drops with terminal velocities of $\sim 1 \text{ m s}^{-1}$ and corrected the lower bound of his Doppler spectra accordingly. But Atlas points out that, in the precipitation streamer shown in Fig. 8.26, the drops may have been sorted by the action of wind shear and so, in the absence of an independent estimate of the falling speed of the smallest drops actually present, the computed downdraughts may be in error. Of course, once the rain has reached the ground, the observer can determine the maximum sizes and fall speed

radars and two conventional radars with RHI displays, to reveal two-dimensional patterns of both horizontal and vertical air motions and the growth of precipitation within a shower. The location of the radars relative to the position of the shower as it passed over the vertically-pointed Doppler radar, B , is shown in Fig. 8.27. Doppler A was operated with its aerial pointing along the azimuth of B at a sequence of elevations α between 1° and 45° , each sequence taking about 1 min to complete. A velocity corresponding to the peak of the Doppler spectrum, V_p , was estimated for each range gate from the intensity-modulated display of radar A .

$$\text{Now,} \quad V_p = -U_\theta \cos \alpha + W_p \sin \alpha, \quad (8.28)$$

where U_θ is the component of the horizontal wind along the azimuth of the beam (positive away from the radar) and W_p the vertical velocity corresponding to the peak of the spectrum (positive downwards). Since W_p was known vertically above B , it was possible to calculate the horizontal wind component U_θ above B , and data from a sequence of elevation scans permitted the construction of height-time sections of U_θ above B . The component of the wind along AB relative to the shower is

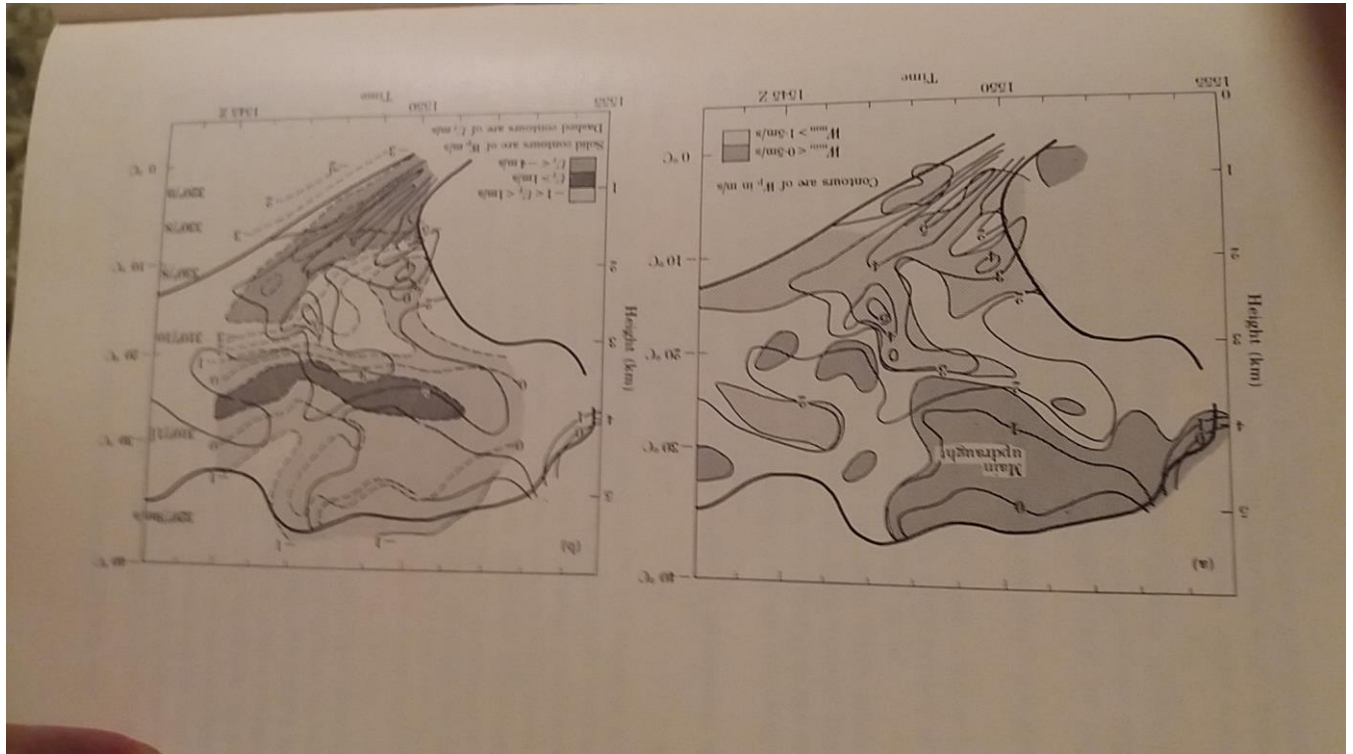
$$U_i = U_\theta - U_s, \quad (8.29)$$

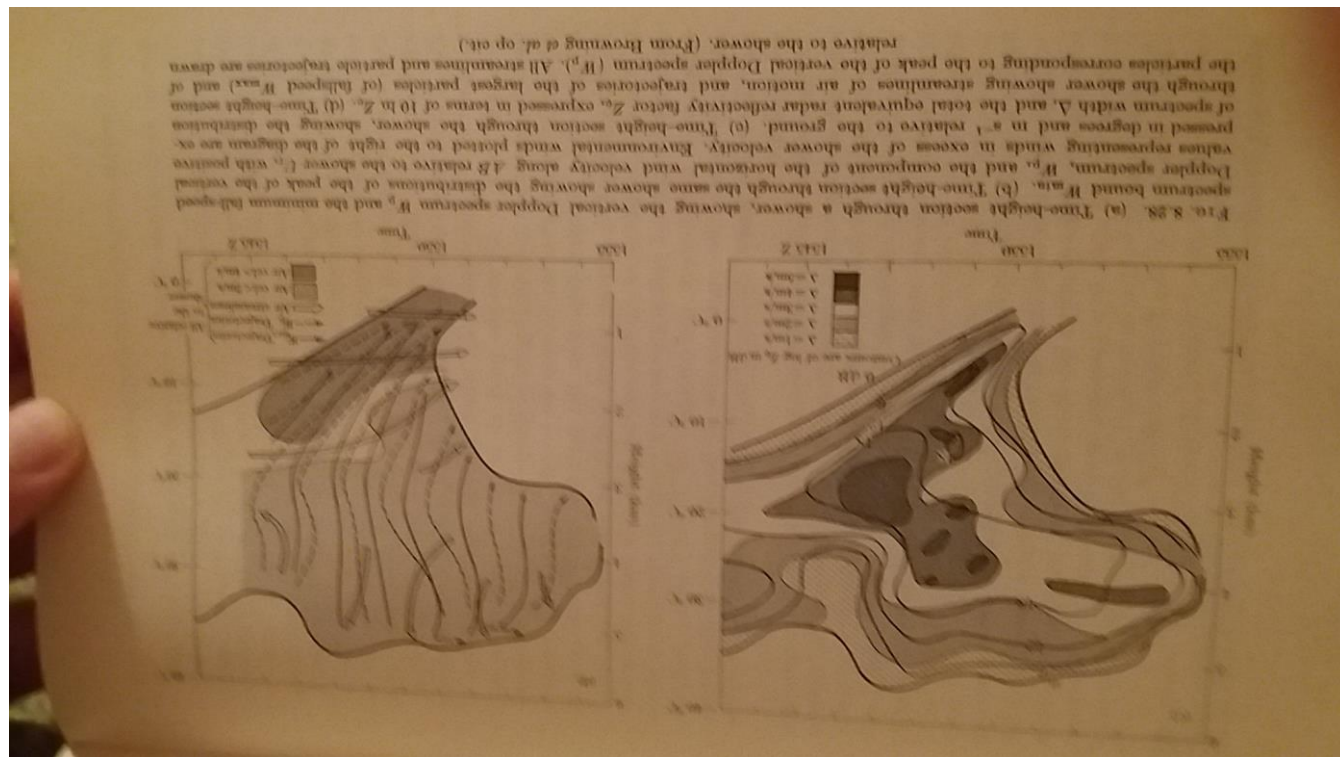
U_s being the component of the horizontal motion of the shower along AB as measured by the conventional radars.

Fig. 8.28(a)–(c) show height-time sections in which the time scale may be interpreted as horizontal distance on the assumption that the shower maintains a steady state during its passage over the vertically-pointing Doppler radar. The diagrams, on which the time scales have been reversed to put the front of the storm on the right, show

- (a) contours of the vertical velocity, W_p , and areas shaded to show where the minimum Doppler velocity, $W_{\min}$, was $< 0.5 \text{ m s}^{-1}$ and $> 1.5 \text{ m s}^{-1}$;
- (b) contours of W_p , and of the component of the horizontal wind, U_i , along AB relative to the shower;
- (c) the distribution of the Doppler spectrum width (in this particular case, almost synonymous with the intrinsic fall-speed spectrum of the hydrometeors), and contours of total radar reflectivity Z_e .

Fig. 8.28(c) shows that both Z_e and the spectrum width were much reduced above the -30°C level where the particles were probably ice crystals having terminal velocities of about 0.75 m s^{-1} . Estimates of the





vertical motions were therefore made by subtracting 0.75 m s^{-1} from the values of W_{600} , and revealed the existence of only weak vertical motions, a mild updraught of only about 1 m s^{-1} above 4 km, and a fairly extensive downdraught of $1\text{--}2 \text{ m s}^{-1}$ below 2.5 km. The light shading of Fig. 8.28(b) denotes areas where U_r was within $\pm 1 \text{ m s}^{-1}$ of the horizontal velocity of the shower, most of the updraught being located in this area. In a small region near the 3.5-km level, the relative wind was towards the front (right) of the shower, but at lower levels the relative winds blew towards the rear, particularly along the axis of the streamer (depicted in Fig. 8.28(b) by the W_p contours), where they reached 4 m s^{-1} . This resulted in the streamer being strongly inclined. Fig. 8.28(c) shows that the distribution of spectrum width was similar to that of the reflectivity, Z_o , which reached maximum values of $\sim 10^3 \text{ mm}^6 \text{ m}^{-3}$ in the middle parts of the shower, and shows that particle growth, as indicated by the gradients of Z_o , occurred principally within the updraught area between the 4- and 5-km levels. The contours of W_p in Fig. 8.28(a) show that the particles contributing most to Z_o were descending in most places, with fall-speeds increasing from 2 to 5 m s^{-1} ; Browning *et al.* demonstrate that this behaviour is consistent with soft hail pellets growing to a maximum diameter of about 6 mm in a liquid-water concentration of 1 g m^{-3} —about half the adiabatic value. The predominance of downdraughts below 2.5 km may have been due to chilling of the air by partial evaporation of the precipitation.

The vertical and horizontal motions were combined to give the streamlines of air motion and particle trajectories shown in Fig. 8.28(d).

This draws attention to the air-flow relative to the shower, and to the trajectories of both the largest and 'average size' particles feeding the precipitation streamer that produced a shower of hail and rain at the ground.

8.6.3. *Determination of horizontal winds, wind shear and convergence*

As the beam of a Doppler radar is tilted down from the vertical, the radial motion of the particles has an increasingly large component due to the horizontal wind. In a horizontally uniform wind field of velocity V_h at the sampling height, as indicated in Fig. 8.29(a), the mean radial velocity measured by the Doppler radar is

$$V_d = V_r \sin \alpha + V_h \cos \beta \cos \alpha, \quad (8.30)$$

where α is the elevation angle of the beam, β the azimuth angle of the

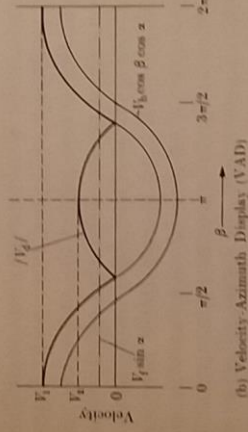
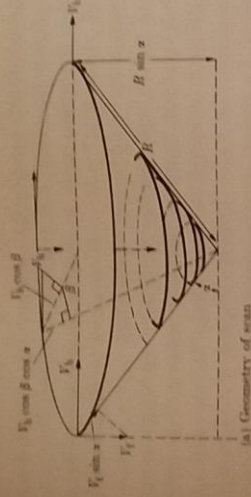


FIG. 8.29. (a) Geometry of scan for wind measurements by VAD techniques. (b) Wind and particle fall speed make up VAD pattern as indicated. (After Lhermitte and Atlas (1961).)

beam with respect to the upwind direction, and V_r is the mean fall speed of the particles relative to the ground. The contributions of both V_r and V_h to the Doppler velocity V_d as a function of β are illustrated in Fig. 8.29(b). When the sign of V_d is not presented, the radar displays only its magnitude, and the dashed negative portion of the curve is folded over as shown. The method, first developed by Lhermitte and Atlas (1961), has been called VAD for velocity-azimuth display. As will be seen from Fig. 8.29(b), the wind speed is obtained from the average of the two maxima and its direction is that of the major maximum. The particle fall speed is obtained from the difference between the two maxima. So

$$V_k = (V_i + V_o)/2 \cos \alpha, \quad (8.31)$$

$$V_i = (V_1 - V_2)/2 \sin \alpha. \quad (8.32)$$

The method has been employed by Lhermitte and Atlas (1961), who estimated that the wind speed can be measured to better than $\pm 1 \text{ m s}^{-1}$ and its direction to within a few degrees. However, small residual

continuity equation. These wind data could, of course, be related to the pattern of radar reflectivity.

Complete three-dimensional information on the wind field and determination of particle falling-speeds at several levels would require the simultaneous use of three Doppler radars (see Lhermitte (1968)).

8.7. The structure of precipitating layer clouds as revealed by radar

8.7.1. General features

The echoes received from widespread, steady precipitation normally associated with warm fronts and depressions lend themselves more readily to quantitative study than those from shower clouds, where conditions are changing very rapidly and interpretation becomes much more difficult. Typical photographs of the RHI and A-scope displays of warm-front precipitation are shown in Figs. 8.17 and 8.20. We have already mentioned the 'melting band' in the vicinity of the 0°C level, which separates a relatively steady echo obtained from the snow above the melting region and a rapidly fluctuating echo of often rather greater intensity from the rain below. The appearance of the melting band, which was accounted for by the theoretical calculations of Ryde (1946), can be explained qualitatively as follows. Above the 0°C level the scattering particles are snow crystals and snowflakes, growth and aggregation of which cause the echo intensity to increase with decreasing height. As the snowflakes fall through the 0°C level, they start to melt, and since the reflectivity of a water particle is greater than that of an ice particle of equal volume, this melting will be accompanied by an increase in the echo intensity, which may be enhanced further by aggregation and preferred orientation of the wet snowflakes. It is usually assumed that, during the melting process, the particles retain the relatively slow fall velocities characteristic of snow, but once completely melted, they collapse to form raindrops having considerably larger fall speeds. As the precipitation rate is usually assumed to remain constant at all levels, this means that the spatial concentration of raindrops will be reduced relative to that of the parent snowflakes in the ratio of their respective terminal velocities. Thus just below the level where complete melting occurs, the echo intensity will show a sharp decrease, with the result that a narrow band of enhanced echoes up between this level and the 0°C isotherm. This interpretation of the melting band has been confirmed by combined aircraft and radar observations (e.g. Bowen 1951).

We shall now discuss in greater detail the echo intensity and its changes with height in the region above the 0°C level, in the melting region, and also in the rain below the melting zone.

8.7.2. The region above the 0°C level

Well above the 0°C isotherm, the scattering particles are mainly snow crystals. In stratiform clouds, where the liquid-water content is generally small, it is likely that the crystals will grow more rapidly by deposition of water vapour than by accretion of cloud droplets. Their growth rate by both processes can, in principle, be computed in terms of the temperature, pressure, humidity, and liquid-water content of the cloud, and hence the back-scattered power, $P_r \propto \sum nm^2$, can be calculated as a function of height in the cloud, if a concentration and relative size-distribution function are assumed. In practice, such computations involve making rather bold assumptions concerning the density and shape of the growing crystals, the magnitude of the vertical air currents in the cloud, the effect of the cloud droplets on the crystal growth-rate, and whether or not a steady-state may be assumed; thus the answers obtained depend on what assumptions one regards as reasonable. Some calculations by Browne, Palmer, and Wormell (1954) indicated that Browne's measurements of echo intensity as a function of height could not be interpreted in terms of ice crystals growing by sublimation in a cloud in which the water budget is balanced.

Such computations of relative echo intensities at different levels, in which a steady state was not assumed, were made by Wexler (1952) for a model cloud following the 10°C saturated adiabatic, with base at 0°C and 805 mb, and sustained by a constant updraught of 10 cm s^{-1} . He first calculated the growth of ice crystals originating at the 3-km level and growing solely by diffusion of vapour in a water-saturated atmosphere, using equations analogous to (5.1)–(5.3). They were assumed to grow as plates with constant thicknesses of $x = 15$ and $40\text{ }\mu\text{m}$ respectively. The results are shown in Table 8.4.

At a given level, all crystals were assumed to have the same mass, and the crystal concentration was taken to be constant with height, so that the ratio of echo intensities at two levels is then $R = m_1^2/m_2^2$, where m is the crystal mass. Calculated values of R relative to the 2-km level are compared with the measured values obtained by Browne (1952a) and by Hooper and Kippax (1950a) in Table 8.5.

Because the calculated values of R in Table 8.5 increase much more rapidly than the observed values, and because the estimated values

TABLE 8.4

Masses and terminal velocities of crystals originating at 3 km above base of model cloud (Wexler 1952)

Height above 0° C (km)	15 μ m crystal		40 μ m crystal	
	Mass (mg)	Velocity (cm s ⁻¹)	Mass (mg)	Velocity (cm s ⁻¹)
2.0	0.65	41	0.12	72
1.5	1.35	41	0.20	72
1.0	2.08	40	0.29	70
0.5	2.68	40	0.36	70
0	2.96	40	0.39	70

masses are much larger than those normally observed, Wexler concluded that, in clouds with small updraughts, ice crystals do not grow at water saturation except, perhaps, in the region below 1 km where there is quite good agreement between the observed and calculated ratios of echo intensities at 1 km and 0.5 km. In consequence, he considered the case where all the water created by the updraught within each height interval is deposited directly on the growing crystals within that interval, so that there is no storage of liquid water in the form of cloud droplets. Taking the upper boundary of the cloud at 4 km and $\rho_a U$ to be constant with height, where ρ_a is the air density and U the updraught speed, the value of R is now given by $R = (q - q_1)^2 / (q_2 - q_1)^3$, where q is the specific humidity. Values of R thus calculated for Wexler's cloud model are shown in the last column of Table 8.5; these are in much better agreement with the observations, suggesting that the crystals grow in conditions intermediate between ice- and water-saturation. However, it must be said that the calculated values of m

TABLE 8.5

Mean observed and theoretical ratios of echo intensity relative to 2-km level (Wexler 1952)

Height (km)	Hooper and Kippax	$R = m_1^2 / m_2^2 \dagger$		$R = \left(\frac{q - q_1}{q_2 - q_1} \right)^3$
		$z = 15 \mu\text{m}$	$z = 40 \mu\text{m}$	
2.0	1.0	1.0	1.0	1.0
1.5	2.2	1.9	2.8	1.9
1.0	3.1	2.0	5.8	3.1
0.5	4.7	4.6	9.0	4.7
0		8.3	10.6	7.2

$\dagger$ Corrected values

and R in Tables 8.4 and 8.5 depend a good deal on the assumptions made concerning the geometry of the growing crystal. Thus Browne (1952a), who assumed that instead of remaining practically constant, the thickness of a crystal was always one-fifth of its radius, concluded that the crystals would not grow fast enough to account for the observed echo intensity, even at water saturation, unless additional growth by accretion of supercooled droplets was postulated—a deduction directly opposed to that made by Wexler.

Calculations by Wexler and Atlas (1958) of a steady-state water budget for a stratiform cloud, which follows a saturated adiabatic, has a parabolic distribution of vertical velocity, and contains particles with an exponential size distribution, predicted that, while production of liquid water is likely to exceed consumption just above the 0°C level, at temperatures below -2°C , the rate-of-growth of ice crystals by deposition is such that the cloud liquid water vanishes above the -4°C level for a surface precipitation rate of 2 mm h^{-1} , and above the -10°C level with 10 mm h^{-1} . The radar observations of Biswas, Murty, and Roy (1960), indicating only a very small increase of echo intensity between the 2 km and 1 km levels above the 0°C isotherm, suggested that the snow crystals were growing slowly in air only slightly supersaturated with respect to ice in the majority of their stratiform clouds.

Although aggregation of snowflakes probably occurs much more readily in the melting region, snow reaching the ground with temperatures at, or a few degrees below, freezing, generally contains aggregates consisting of several individual crystals, and suggests that aggregation may also occur above the 0°C level in the cloud.

If n individual crystals join together to form a cluster having a fall velocity f times that of an individual crystal, the echo intensity will be increased n/f times. Lhermitte and Atlas (1963), using a Doppler radar, deduced that the reflectivity of dry snowflakes increased sixfold, and their average terminal velocity doubled, in falling through a vertical distance of 750 m just above the melting zone. Allowing for some growth by deposition of vapour and droplets on the crystals, the authors interpreted these results in terms of an initial population of uniform-sized crystals forming aggregates of nine crystals during the interval. But it is generally difficult to locate the 0°C level with sufficient accuracy to delineate the freezing from the melting region, and so separate the effects of aggregation that may occur at temperatures just above and below 0°C . This should also be borne in mind

when we come to discuss measurements of signal intensity in the melting zone.

8.7.3. *The melting region*

Measurements designed to locate the height of the melting band relative to the 0°C level, to determine its vertical thickness, and the radar reflectivity in the band relative to that in the snow above and the rain below it, have been made by Austin and Bemis (1950), Hooper and Kippax (1950b), and Browne (1952a). Difficulties arise because the measured range and width of the melting band depend on the pulse duration and the band-width of the receiver. Hooper and Kippax investigated the relationship between the heights of the melting band and the 0°C level using a vertically directed 3.2-cm radar based at a radio-sonde station, radar observations being made simultaneously with radio-sonde ascents from which the heights of the 0°C level were obtained. The results of observations made on ten occasions showed that, on average, the maximum of the melting band lay 330 ± 150 ft below the 0°C level. The melting-band echo was observed to be about $2.5\ \mu\text{s}$ in duration using a set with pulse duration $1\ \mu\text{s}$, so that the vertical thickness of the band appeared to be about 750 ft. Austin and Bemis (1950), using a RHI display and working at ranges of more than 10 miles, give an average figure of 830 ft for the distance of the melting band below the 0°C level, but they admit of errors, of perhaps 500 ft, in the location of this level and of the melting band. The resolving power of their equipment did not allow them to make good measurements on the thickness of the band, but they separated their observations into occasions with stable lapse rates when the band was less than 1000 ft thick, and occasions with steep lapse rates when the band was more than 1000 ft thick. These authors quote one observation made with a vertically pointing radar with a $0.5\ \mu\text{s}$ pulse, when the apparent thickness of the melting band was about 500 ft. Browne (1952a), using a similar arrangement, found the peak of the melting band to be 600 ± 200 ft below the 0°C level (but his observations included some cases of unsteady rain) and that, in warm fronts, the band was generally 300 to 400 ft thick. An analysis by Mason (1955c) of records obtained by Browne for twelve further occasions of steady rain reveals values between 50 and 500 ft for the depth of the melting zone, the average value being 225 ft. A progressive lowering of the melting band may occur due to the cooling of the air by the melting snow (Wexler, Reed, and Honig 1954); it is also frequently observed to rise due to advection of warm air.

The depth of the melting zone, measured from the 0° C level to where it merges into the rain echo below, corresponds to the distance through which the majority of the snowflakes will fall before melting, and will therefore depend upon the lapse rate and on the sizes and fall velocities of the melting flakes. The author calculates that a flake of mass 1 mg composed of dendritic crystals and falling through an atmosphere with lapse rate 6° C/km, would melt completely after a fall of about 150 m. The results of similar but more extensive calculations have been published by Wexler (1955).

8.7.4. *The region below the melting band*

The echo from the rain below the melting band exhibits marked fluctuations in intensity from pulse to pulse, but the mean intensity changes only very slowly with height. On some occasions, an increase of signal strength with decreasing height is to be expected as the result of coalescence between raindrops and growth of raindrops by accretion of cloud droplets. These processes have been investigated theoretically by Mason and Ramanadham (1954) and Hardy (1963); the former calculated that the raindrops associated with precipitation of intensity 3 mm h^{-1} , in falling a distance of 1 km in a cloud of liquid-water content 0.2 g m^{-3} , should grow sufficiently to increase the radar echo intensity by a factor of about 2. Such increases were observed on a few occasions by Harper (1957) but, in the majority of warm-front situations, he observed no such increase.

8.7.5. *Polarization of the radiation scattered by hydrometeors*

Some information on the shape and orientation of the precipitation particles might be expected from a study of the parallel- and cross-polarized components of the back-scattered radiation. In §8.5.2. it was argued that melting snowflakes, which may be treated approximately as oblate spheroids, will, if randomly oriented, produce a cross-polarized component, which is independent of the direction of polarization and the angle of incidence of the radiation. If they are oriented with their figure axes vertical, no cross-polarization will result, but if the aerial is vertically polarized, the intensity of the back-scattered radiation will increase as the beam moves towards the zenith. Browne and Robinson (1952) found that the particles in the melting region produced back-scattered radiation with a greater cross-polarized component than that given by the snow above and the rain below. The observations were made by using separate aerials for the transmitter and receiver and by comparing the echo received when the aerials had

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their planes of polarization parallel with that received when the plane of polarization for reception was at right angles to that for transmission. Wavelengths of both 3.2 cm and 3 mm were used with the results shown in Table 8.6.

The values for snow above the 0°C level are uncertain because of the very weak echoes, while the values given for rain may have been a measure of the cross polarization of the aerial systems.

Further data were obtained by Hunter (1954) using a 3.2-cm transmitted beam that was circularly polarized. He measured the ratio of the orthogonal circular components, a quantity that is about twice the depolarization ratio for a plane-polarized beam. The most comprehensive measurements have been made by Newell (1956) using a common

TABLE 8.6

Ratio of parallel to cross-polarized components of back-scattered radiation from 'steady' precipitation, i.e. (depolarization ratio) in dB

Author	Rain	Melting zone	Snow
Browne and Robinson (1953)	15 ± 1	7 ± 1	12 ± 2
Hunter (1954)	21.9 ± 0.4	19.9 ± 0.3	19.9 ± 0.6
Newell (1956)	26-36	16-23	29
	19-20	16-22	16-20

transmitting and receiving antenna that was first plane-polarized and then circularly polarized, the ratio of the intensities of circularly to plane-polarized signals being twice the depolarization ratio if the particles are randomly oriented. Inspection of Table 8.6 reveals considerable discrepancies between the results of Hunter and those of the other workers. The latter indicate that raindrops and dry snowflakes may show considerable departures from sphericity, the distortion being even greater for melting snowflakes. Hunter finds considerable smaller departures from sphericity in all cases, the axis/diameter ratios for raindrops rarely being less than 0.85. Newell found that rain and snow particles were randomly oriented as far as the radar could detect, but that, on occasions, particles in the melting zone showed some tendency towards horizontal orientation.

8.7.6. *The echo intensity as a function of height*

The intensity of the echo from the centre of the melting zone relative to that from the rain below and that from the snow crystals above has

The behaviour of the South African, Australian, and Canadian clouds was very similar in that all three sets of trials indicated that the chance of inducing precipitation by the introduction of dry ice is high if the cloud top is colder than -7°C and the depth of the supercooled region exceeds 4000 ft. These results were confirmed and amplified by Bowen (1952a) in a review of the Australian work in which, on the basis of about 100 dry-ice experiments performed in the period 1947-51, he drew the following conclusions.

(1) With cloud summit temperatures of -7°C and colder, there is a 100 per cent chance of producing precipitation. At temperatures between -7° and 0°C , the chances of success fall off progressively, tending to zero at 0°C . At temperatures of approximately -15°C and below, the results lose their significance because of the high probability of clouds producing rain naturally.

(2) The time at which precipitation will appear at the base of the cloud depends mainly on the thickness of the cloud; there is a gestation time of about 10 min, followed by an additional time of about 1 min for every 800 ft. of cloud.

(3) The intensity of the precipitation from the base of the cloud increases with increasing cloud thickness.

(4) A considerable fraction of the precipitation will reach the ground if the height of the cloud base is equal to, or less than, the cloud thickness.

However, the summit temperature and cloud depth cannot be the only factors that will determine the chances of successful seeding; for example, the strength and duration of the updraughts must be sufficient for the particles to grow to precipitation size. Herein may lie an explanation for the fact that a similar programme of dry-ice seeding of supercooled cumulus clouds in the central United States, described by Braham, Battan, and Byers (1957), produced no evidence that the dry ice initiated precipitation.

Although most of the above experiments may be criticized for not being randomized with a view to eliminating observer bias (see p. 384), they do indicate that, under some circumstances, seeding cumulus clouds with dry ice may initiate or enhance precipitation, and in others it may inhibit cloud growth and decrease precipitation. In only rather rare cases was seeding followed by more than a light shower at the ground.

7.2.2. Seeding cumulus with silver iodide

There are rather few data on the effect of seeding *individual* cumulus clouds with silver iodide. In order to infect the region bounded by the

5 and -10°C inside in a cloud of cumulonimbus near 5 km² with clouds in an average concentration of 1/1000, would require about 10¹⁰ nuclei, so that the required quantity of silver iodide, if this were introduced from an aircraft, should be only a few grams.

Warner and Tammey (1966) describe an experiment conducted in Australia in which 35 suspended cumulus clouds, with vertical temperature ranging from -2.5 to -10°C and depths ranging from 4000 to 15 000 ft, were seeded with about 10 g (or 10⁴ nuclei) of silver iodide from an aircraft flying through the upper levels of the cloud or just below the cloud base. Of these clouds that were colder than -5°C at their summits, 72 per cent precipitated usually within 20–25 min of seeding, 21 per cent evaporated, and 7 per cent showed no discernible effects. As first night, these results seem very similar to those described above for seeding with dry ice, but further trials, in which clouds were selected for seeding on a randomized basis, have failed to produce such clear cut results.

Bedrossian, Smith, Wurster, and Moftakian (1966) describe trials involving 60 isolated suspended cumulus clouds in which either a large (20 g), small (0.2 g), or zero quantity of silver iodide was introduced into the cloud base, with random choice of treatments. A cloud was regarded as suitable for treatment if its summit was either colder than -5°C or judged likely to become colder than -5°C within 20 min, if its depth was >1000 m and exceeded half the terrain clearance, if the height of the base did not exceed 2500 m, if the cloud was fairly isolated, dense, and compact with a well-defined base and top, and was not inclined in a strong wind shear. A further condition was that neither the selected cloud nor any other within a radius of 20 km should show visible signs of either precipitation or glaciation. The quantity of rain that fell from the cloud was estimated from the numbers and sizes of raindrops recorded on a fall impactor sampling device flown beneath cloud base. The authors claim that the 11 clouds with tops colder than -10°C which were treated with the larger quantity of silver iodide yielded significantly more rain than similar, untreated clouds. However, in view of the very small sampling volume of the impactor device, the large sampling errors, and the fact that the total rain falling from a cloud was based on a rain sample of only a few cm³, this claim is not at all convincing.

A particularly well conceived experiment designed to evaluate the effect of silver iodide seeding upon individual cumulus clouds has been described by Simpson, Brier, and Simpson (1967), who regarded it

The Physics of Natural Precipitation Processes

PRECIPITATION, which may reach the ground in the form of rain, snow, or hail, is one of the most marked characteristics of weather and is the end product of the physical processes described in this book.

The occurrence of precipitation is strongly controlled by the motion of the cloud air. We have seen in previous chapters how the air motion during the formation of the cloud, in combination with the properties of those aerosols it contains and which act as condensation and freezing nuclei, determines the concentration, initial size distribution, and nature of the cloud particles. As soon as these have formed, the microphysical processes of condensation and aggregation begin to deform the spectrum, the production of precipitation elements now being under way. Because the air motion governs the dimensions, water content, and duration of the cloud, it controls not only the rates of these processes, but the period during which they operate and thus the maximum size that the largest of the cloud particles can attain. If precipitation should result, it is again the air motion that determines its distribution, intensity, and duration. Conversely, the growth and evaporation of particle populations, accompanied by changes of phase and water concentration, provide sources and sinks of heat that can profoundly influence the air motion. Thus the release of latent heat during the growth and freezing of cloud droplets, by providing additional buoyancy to the cloud, may promote and sustain its growth until the updraught is destroyed, either by mixing with the drier surroundings, or by accumulation of condensed water and the formation of a downdraught sustained by evaporation of the precipitation.

The factors of prime importance therefore appear to be the air motion and its aerosol content, and from specifications of them it should be possible, in principle, to calculate quantitatively the course of cloud and precipitation development. However, promising techniques for the observation, measurement, and computation of the air motions in and around clouds are only beginning to be developed, and the laws governing the formation, growth, and aggregation of cloud and precipitation particles are not yet fully established. Moreover, the great

variations in the concentration and properties of atmospheric aerosol, and the great complexity of atmospheric motions (as revealed by the great variety in the scale and pattern of cloud development), add to the difficulty of constructing a general quantitative theory of precipitation development. As a result, little progress has been made in formulating realistic dynamical models of cloud systems, which are usually influenced by motions on scales both larger and smaller than the system itself. So far, we have not progressed much beyond constructing simple kinematic models to follow the history of condensed water and precipitation in a specified (rather than derived) field of vertical motion, and the construction of simple one-dimensional models of cumulus growth that allow the vertical motion to be *computed* in terms of buoyancy forces modified by the formation of condensed water, its partial conversion to precipitation, and the growth, fall-out, and evaporation of hydrometeors. These models, in which both the cloud physics and the interaction between the cloud and its surroundings are only rather crudely represented by semi-empirical relationships based on limited observational or laboratory data, have proved useful in illustrating the relative importance of various factors that can influence the maximum vertical development of the cloud and the initiation and spread of a downdraught, but they are inadequate to predict the intensity, distribution, and duration of precipitation within and beneath the cloud. This must await the development of detailed three-dimensional, time-dependent models, in which mixing and interaction with the environment arises as a direct consequence of the computed three-dimensional wind field and of turbulent mixing. Here we may note the encouraging progress made by Bushby and Timpson (1967) and Benwell and Timpson (1968) in the numerical prediction of rainfall associated with fronts and depressions, with scant attention to the cloud physics, but the formulation of realistic three-dimensional dynamical models of a cumulonimbus, involving strong interactions between the precipitation and the dynamics, is likely to prove much more difficult.

We shall not penetrate much further into the realm of cloud dynamics, but rather concern ourselves with the microphysical processes involved in the formation, growth, break-up, and evaporation of precipitation elements. A good deal of insight has been gained into the limiting conditions under which hydrometeors of various types may grow in clouds of individual particles in simple, specified updraughts. We shall see that such calculations provide at least a reasonable qualitative

explanation of the development of the various forms of precipitation in different types of clouds.

6.1. Forms of precipitation

We have seen that a non-precipitating water cloud is composed of minute droplets rarely exceeding a few tens of microns in diameter and possessing terminal velocities of only a few centimetres per second. Thus, considerable growth of a cloud particle must occur before it can gain a falling speed sufficient for it to fall out of the parent cloud, survive evaporation in the unsaturated air beneath, and reach the ground as a precipitation element.

Clearly, the size of precipitation elements will be determined, to some extent, by the strength of the upward air current producing the cloud and by the humidity in the sub-cloud layer. Widespread layer clouds are associated with upcurrents of speed usually less than 0.5 m s^{-1} , so that droplets with a radius approaching $80 \text{ }\mu\text{m}$ can fall out of them and approach the ground. Findeisen (1939a) estimated the distance a drop can fall through unsaturated air before completely evaporating, and has shown it to be proportional to almost the fourth power of the initial radius. In an atmosphere of 90 per cent relative humidity, he found the distance to be 3.3 cm when the radius is $10 \text{ }\mu\text{m}$, 150 m when it is $100 \text{ }\mu\text{m}$, and 42 km when the radius is 1 mm. Since (except in special circumstances) the bases of dense clouds lie at least a few hundred metres above the ground, a value for the radius of about $100 \text{ }\mu\text{m}$ may be regarded as a lower limit to the size of precipitation elements.

Precipitation composed entirely of drops little larger than this commonly falls in damp weather from layer clouds not far above the ground, and is called *drizzle*. Drizzle (and, in cold weather, snow crystals or small flakes) may fall from shallow clouds whose thickness amounts to only a few hundred metres. *Rain*, consisting of drops which are larger, having radii up to about 3 mm, is produced by layer clouds some thousands of metres deep, such as are associated with fronts and depressions. The heaviest rains, composed of relatively large drops, fall from convective (cumulus) clouds whose depth may reach 10 km and which contain powerful upcurrents of several metres per second. Precipitation from these clouds is sporadic, for their horizontal extent is only a few kilometres and the active life of individuals is less than 1 hour. These rains are described as *showers*.

Shower clouds that extend well above the level of the 0°C isotherm

are believed to contain ice particles, mainly in the form of *graupel* or *hail pellets*, in their upper regions, but these may often melt before reaching the ground. However, very deep and vigorous clouds may produce very large hailstones which, in hot climates, are occasionally as large as oranges. In cold weather the precipitation may reach the ground entirely as small hail.

Even the largest snowflakes have fall speeds hardly exceeding 1 m s^{-1} , and so *snow* falls only from layer clouds and from weak and decaying convective clouds which contain small vertical air currents.

Ice clouds, even at levels where the low temperature restricts the total amount of water present to less than 1 g m^{-3} , are often composed of particles large enough to have fall speeds of nearly 1 m s^{-1} . This is partly because the rare crystals appear only when the vapour pressure approximates to saturation over liquid water, so that beneath the condensation region there is a layer of air which, although not saturated with respect to liquid water, is nevertheless supersaturated with respect to ice. The crystals therefore continue to grow while they fall through this layer, which is often a kilometre deep and may be much more. In this way, a few crystals draw on the vapour in a great volume of air and become much larger than the droplets of unfrozen clouds. Ice clouds therefore often appear as fibrous streaks known as Fallstreifen or virga, which may be regarded as precipitation.

6.2. Physical processes responsible for release of precipitation

In droplet clouds the condensation process alone is usually ineffective in producing precipitation. In the lower and middle troposphere, where the droplet concentrations established during cloud formation are believed always to be of order 100 cm^{-3} , and the quantity of vapour condensed cannot exceed 7 g m^{-3} , the *average* radius of cloud droplets must be rather less than $30 \text{ }\mu\text{m}$. Droplet concentrations arising during condensation in very clean air at high levels may be one order of magnitude less, but the amount of vapour to be condensed is also greatly reduced, and so the conclusion that condensation alone is unlikely to produce droplets of *average* radius greater than $30 \text{ }\mu\text{m}$ is not affected. Droplets of this size evaporate completely after falling a few metres in unsaturated air, and so are incapable of reaching the ground as precipitation. Condensation on giant hygroscopic nuclei may produce *some* larger droplets but these will attain radii of $100 \text{ }\mu\text{m}$ only after several hours. In the layer-type clouds that have this duration the

vertical air velocities are considerably less than 1 m s^{-1} , so that the droplets would fall out of them before reaching even this size.

The search for an adequate physical explanation of the formation of precipitation elements was apparently concluded with the publication of a classical paper by Bergeron (1935). In this paper Bergeron discussed the following factors that might possibly lead to non-uniformity in an initially nearly homogeneous cloud of water droplets: (a) the attractive forces between cloud droplets by virtue of their having either unlike electric charges or induced charges in the earth's electric field; (b) hydrodynamic attraction forces between drops of nearly the same fall velocity; (c) differences of capillary and hygroscopic forces at the surfaces of droplets of slightly different sizes; (d) temperature differences between drops of different origin leading to distillation of vapour from warmer to colder droplets; (e) the effect of turbulence on causing collisions between droplets. Bergeron concluded that all the above-mentioned mechanisms were inefficient or much too slow, and that the only factor that could account for the release of precipitation was the appearance of a few ice crystals (formed either by the freezing of droplets or, less likely, by the sublimation of vapour on special nuclei) among a much larger population of supercooled droplets in those parts of the cloud where the temperature was below -10°C . In such mixed clouds, the state of approximate water saturation represents a supersaturation with respect to ice of 10 per cent at -10°C and 21 per cent at -20°C , so that rapid growth of the crystals would be expected. The physical mechanism discussed by Bergeron was just that put forward earlier by Wegener (1911), viz. 'The vapour tension will adjust itself to a value in this must then be, that condensation will take place continually on the ice, whereas at the same time liquid water evaporates [from the droplets], and this process must go on until the liquid phase is entirely consumed.' Bergeron postulated that almost every raindrop ($d > 0.5 \text{ mm}$) originates as an ice particle that 'grows copiously' in this way, and therefore that all rain clouds must extend well above the level of the 0°C isotherm. He claimed to have conclusive proof of this in the observed behaviour of cumulus clouds in Europe which, he said, do not produce showers until their summits reach above the 0°C level and then become transformed into fibrous ice clouds.

While direct evidence for the dominance of the Wegener-Bergeron mechanism in precipitating clouds was meagre at this time, it received strong support from Findeisen who, however, believed that the ice

crystals arose by the sublimation of vapour on to special sublimation nuclei rather than by the freezing of droplets. On the basis of sublimation observations made over Germany, Findeisen (1939*b*) stated categorically that all rain composed of large drops originates as snow or hail. Further data on the temperatures of cloud summits (see § 6.3) appeared to support this thesis, so that the Bergeron hypothesis came to be generally accepted during the following decade; in the absence of detailed quantitative arguments, all other possible processes were deemed to be much too slow.

However, it must have been obvious to both meteorologists and aviators that, in tropical regions, showers often fell from clouds whose tops did not reach within thousands of feet of the 0°C level. Rumours to this effect were current from time to time, but for some peculiar reason, very few reports of detailed observations were published and the views of the European meteorologists on the formation of rain were not seriously challenged. The possibility that the larger droplets in a wholly water cloud (which are never found to be as uniform as imagined by Bergeron) might continue to grow by collision with their small neighbours was not entirely dismissed. Indeed, Findeisen (1939*b*) calculated the radii with which drops of initial radius $30\text{ }\mu\text{m}$ would be expected to fall from a layer cloud of liquid-water content 1.25 g m^{-3} and found values of 100, 200, and $400\text{ }\mu\text{m}$ for cloud thicknesses of 150, 400, and 1000 m respectively. However, he rejected this result as contrary to experience, and believed his theory to be in error in that it assumed that a larger falling drop would collect all the droplets in its path, none being swept aside. He was firmly of the opinion that this coalescence mechanism was a slow and inefficient method of precipitation release capable of causing only falls of drizzle—an opinion endorsed by Bergeron (1949) in the case of all extra-tropical clouds. But reviewing the subject in 1941, Simpson (1941) emphasized that the belief that raindrops necessarily originate as ice particles rested on statements that rain was never observed to fall from pure droplet clouds, even when they were several thousand metres thick. Simpson would not accept these statements, and suggested that copious rain could be produced by wholly liquid clouds, providing only that the original droplets were of different sizes so that the coagulation process could start, and that the cloud was of sufficient depth.

In recent years, careful visual and radar observations have accumulated which show beyond all doubt that in tropical regions heavy rain falls from clouds that are entirely beneath the level of the 0°C isotherm.

In these regions this level may be above 5 km whereas in temperate latitudes it is often below 2 km, and here equally deep clouds commonly reach heights where the temperature is below -20°C , so that their tops would almost certainly contain ice crystals. It does not follow, however, that their presence is essential to the production of rain, and there is increasing evidence that showers in temperate latitudes are often *initiated* by the coalescence process.

A more detailed evaluation of droplet growth by coalescence in wholly water clouds was made possible by the publication of a paper by Langmuir (1948) in which he calculated the efficiency with which a drop of given size would collect smaller droplets lying in its fall path (see Appendix A). He went on to calculate the growth of large water drops by this coalescence process following their introduction into the upper parts of convective clouds. He did not suggest that shower rains could arise naturally by this mechanism alone, but thought that melting hail might be an important agent in introducing large drops into the lower parts of shower clouds.

Houghton (1950) used Langmuir's values of collision efficiency to compare the growth rates of drops by coalescence with those of ice crystals by sublimation. He concluded that the growth of precipitation elements by sublimation is initially much more rapid than by coalescence, the two processes becoming equally effective when the particles are of a mass comparable to drizzle drops, and that for larger particles, the accretion process becomes dominant.

More detailed treatments by Bowen (1950) and Ludlam (1951*a*) start from the assumption of the presence, near the cloud base, of a few particles larger than the main population of cloud droplets and of the occurrence of a significant updraught throughout a considerable depth of cloud. Bowen, who assumed these larger drops to arise by chance combinations of droplets in the lower parts of the cloud, calculated their subsequent growth by condensation and coalescence in clouds of constant updraught and constant liquid-water content. He demonstrated that the process may reasonably result in the production of raindrops in clouds several thousands of feet deep, and described observations which show that, in Australia, showers frequently develop by this process.

The suggestion was made by Woodcock (1950*b*) that the larger droplets required to initiate the coalescence mechanism might be supplied by the giant salt nuclei found by him (see Chapter 3) in concentrations of order 1 litre in the sub-cloud layer over the sea, and which originate

as spray droplets of radius 20–30 μm . It was shown by Ludlam (1951*a*), that in convective weather over the sea, the spray droplets may be carried up to the cloud without appreciable evaporation and thereafter be capable of releasing a shower.

A larger droplet, whatever its origin, will be carried up in the ascending air, but will be falling relative to the ordinary cloud droplets and so will collide and (presumably) coalesce with them. The drop thus grows at an ever-increasing rate and, ultimately, if the updraught extends to a sufficient height, its rate of fall will exceed the speed of the updraught and the drop will return to the cloud base, growing as it descends. The results of the calculations presented in § 6.5 indicate that this process could produce raindrops from convective clouds a few kilometres deep and sustained by updraughts of a few metres per second. In temperate regions, this will usually mean that the top of the cloud will extend above the 0°C level and so may contain ice crystals. It may therefore be a difficult matter to decide which of the two possible mechanisms is responsible for the initial release of precipitation, since both may be active.

Once an ice crystal growing in a dense supercooled cloud has become appreciably larger than a cloud droplet, its growth will be greatly accelerated by accretion of these droplets, which will freeze on impact to form ice pellets of irregular shape. This was pointed out by Findeisen, the implication being that, in convective clouds, the Bergeron sublimation process would be of importance only in the early development of a precipitation element. Some computations on the growth of such ice pellets in model clouds have been made by Ludlam (1952). His calculations suggest that, in clouds with bases warmer than 10°C , and containing updraughts of less than 5 m s^{-1} , droplets of radius 20–30 μm , introduced into the base of the cloud, may grow to precipitation size earlier than will ice particles originating at temperatures below -10°C ; in colder clouds, the ice particles will become favoured.

The growth and structure of large hailstones now find satisfactory quantitative explanations in terms of their accretion of supercooled water and dissipation of latent heat while being lifted and re-cycled in the strong, steady, tilted updraughts of severe storms, largely as the result of important contributions by Schumann (1938), Ludlam (1950, 1958), Browning and Ludlam (1962), Browning, Ludlam, and Macklin (1963), List (1960, 1963), Browning (1966), and Bailey and Macklin (1968*a, b*), all of which are discussed in detail in § 6.6.

The growth of drops by coalescence in layer clouds containing

updraughts of only a few centimetres per second was investigated by Mason (1952a). He showed that, because these clouds generally exist for much longer than shower clouds, the larger droplets necessary to initiate coalescence may develop *in situ* by condensation, but that they cannot grow to much larger than drizzle size by coalescence, unless water contents considerably larger than those usually measured are assumed. Thus it appears that the rain that falls from the thick layer-cloud systems of extra-tropical regions is probably initiated almost entirely by the growth of ice crystals which, however, must aggregate to form snowflakes if they are to produce raindrops of the observed size. The predominance of the Bergeron process in precipitating layer clouds is borne out by radar studies which reveal, almost invariably, the presence of a strong band of echo (the melting band) just below the 0° C level and which is associated with the melting of snowflakes (see Chapter 8). Radar can be used to provide statistics on the frequency of rainfall associated with this process, since the presence of the melting band implies the absence of updraughts stronger than 1 m/s, which are necessary for the formation of rain by the coalescence process.

In this section we have traced, in outline, the development of modern ideas on the release of precipitation. The theoretical arguments will be presented in more detail in § 6.4 and § 6.5, but first it will be as well to review the observational evidence against which the theories must be judged.

6.3. Observational data on the characteristics of precipitating clouds

Three parameters that will obviously have an important bearing on which particular mechanism is likely to be dominant in the release of precipitation from a particular cloud or cloud system are the temperature of the cloud base, the cloud thickness, and the temperature of the cloud summit. The temperature of the base will largely control the liquid-water content of the cloud which, together with the cloud thickness, will largely determine whether or not raindrops may be produced by coalescence, while the cloud-top temperature will indicate the likely presence or absence of ice crystals. These three parameters can be readily measured from an aircraft fitted with a suitable thermometer and altimeter and, in the case of cumulus clouds in the general vicinity of a radio-sonde ascent, from theodolite readings taken on the ground. Unfortunately, reliable observational data of this kind are rather scanty, but those which are available will now be summarized.

6.3.1. *Observations on clouds in middle latitudes*

Aircraft observations of layer clouds in middle latitudes, as reported, for example, by Munro (1940), Stickley (1940), and Peppler (1940a) reveal that although rain may occasionally fall from non-freezing layer clouds, it is usually initiated by the ice-crystal process. On the other hand, drizzle may often fall from clouds that are warmer than 0°C and contain no ice crystals, and is therefore produced by the coalescence of droplets. Of nearly 1000 clouds analysed by Peppler whose tops were reached, and in which ice particles were detected, 20 per cent had summit temperatures higher than -12°C , nearly 50 per cent above -16°C , and 70 per cent above -20°C . This, together with the fact that of the supercooled clouds in which no crystals were detected, about 30 per cent were colder than -12°C , only 12 per cent colder than -16°C , and only 5 per cent colder than -20°C , suggests that, on average, a sharp rise in the production of ice crystals occurs in the temperature range -12 to -16°C . In only 22 per cent of the clouds containing ice crystals was icing encountered at temperatures below -8°C , but it was observed on 63 per cent of occasions at temperatures between 0°C and -4°C .

Apart from the abundance of ice crystals, the probability of the development of precipitation will depend on the cloud thickness, the height and temperature of the cloud base, the magnitude of the vertical air currents, and the lifetime of the cloud, all of which will be governed by the large-scale dynamical and thermodynamical processes. Continuous precipitation can be maintained only if the cloud water is replenished by a persistent vertical motion fed, perhaps, over a large region, by low-level convergence in the wind field.

Peppler reported that about 70 per cent of the snow clouds had thicknesses of 1–4 km and bases below 1.2 km and were warmer than -5°C . On the other hand, all the clouds producing drizzle were warmer than -5°C and most of them were less than 2 km deep.

Peppler's observations were confirmed by Mason and Howorth (1952) in a detailed analysis of the data obtained from aircraft ascents made thrice daily to a level of 400 mb during 1942–4 over Northern Ireland. Of the clouds producing rain or snow whose tops were actually reached, only 28 per cent were warmer than -12°C , but 55 per cent were warmer than -20°C . In only 4 of the 99 well-substantiated cases of rain was the cloud-top temperature above 0°C , and in only 8 cases was it above -5°C . Of these 8 cases, 3 produced moderate and 5 only light rain. The only case of continuous moderate rain was from a layer with

base temperature 16°C , top temperature 5°C , and thickness 8500 ft (2.6 km). All the precipitating clouds had bases warmer than -5°C . Of the 45 reported cases of drizzle reaching the ground, in 20 (58 per cent) the cloud-top temperature was above 0°C , and in 39 cases (87 per cent) it was above -5°C . Clouds that produced snow at the ground were, in every case, colder than -12°C at their upper boundaries. Rain/snow fell only from clouds of thickness greater than 1 km, and continuous rain only from those of depth greater than 2.5 km, while no precipitation reached the ground from clouds less than 500 m in depth.

Rather similar results emerge from an analysis by Stewart (1964) of aircraft ascents made over an 8-year period over south-western England. Of 64 cases of layer clouds with their bases < 1 km above the 0°C level (i.e. cloud base temperature, T_B , above -5°C), 66 per cent gave precipitation at the ground, whereas of the 135 cases with bases more than 1 km above the 0°C level ($T_B < -5^{\circ}\text{C}$), only 5 per cent gave precipitation at the ground. If only clouds of thickness greater than 2 km were considered, the corresponding figures were 88 per cent, and < 4 per cent, respectively. Calculations indicated that snow crystals were unlikely to survive evaporation in falling more than a kilometre. No precipitation fell from clouds < 400 m deep, and there were no cases of continuous slight or continuous moderate rain unless the thickness exceeded 1 km and 3 km respectively. Similar results were obtained for clouds whose bases lay below the 0°C level; at least 60 per cent of these clouds were entirely below this level, were unaffected by higher clouds, and therefore must have produced precipitation by the coalescence process.

We turn now to observations on *shower clouds* in middle latitudes. Peppler (1940*b*), from flights into 79 clouds producing showers over Germany, reported that 75 per cent had summit temperatures below -12°C and there was none lying entirely below the 0°C level. The bases of clouds that ultimately precipitated were warmer than 0°C on 90 per cent of occasions, the average temperature being 7°C . Of the non-precipitating cumuli, 99 per cent never reached the -12°C level, their average base temperature being 2°C .

Findeisen (1942*b*), from a rather small number of aircraft observations made over the North Atlantic, inferred that the summit temperatures of maritime cumulus clouds containing ice crystals, though varying considerably from day to day, were, on average, 6 – 8°C higher than those reported by Peppler for clouds occurring over the land.

However, it seems that, in these observations, glaciation was sometimes inferred from the external appearance of the cloud—a most difficult and uncertain procedure, since the presence of large drops in the upper regions of the cloud can produce the fibrous appearance that is generally associated with the presence of ice crystals. That showers falling over the sea tend to be associated with appreciably warmer clouds than is the case over land, also emerges from the work of Schwerdtfeger (1948), who analysed the observations made during 823 flights over the ocean during 1942–4 near 60° N, 13° W. Rain or snow showers were observed on 250 occasions and the temperatures of the cloud summits determined; 8 per cent of all the shower clouds were warmer than 0° C, and 45 per cent warmer than –12° C. These proportions were, however, considerably higher in the summer months, being 20 and 64 per cent, respectively. These statistics suggest very strongly that showers may be produced by the coalescence process in the maritime cumulus of middle latitudes, particularly in the summer when the cloud-base temperatures are relatively high.

Braham, Reynolds, and Harrell (1951), observing summer cumulus in New Mexico by double theodolites and radar, found that radar echoes never developed when the minimum summit-level temperature, was higher than –12° C. At temperatures between –12 and –24° C, only about one-fifth of the clouds produced echoes, and though the fraction was larger at lower temperatures, a few clouds gave no echoes even at temperatures below –30° C. The cumulus in this region have high bases, often above 10 000 ft (3 km), so that even in summer the base temperature is generally not greater than about 10° C. Of the non-precipitating cumulus studied in New Mexico, 75 per cent were colder than –12° C and 25 per cent colder than –20° C, in striking contrast to those forming over Germany of which only 1 per cent were colder than –12° C.

These geographical differences in cloud behaviour are further underlined by the work of Battan and Braham (1956), who used aircraft equipped with radar to make a census of both precipitating and non-precipitating cumulus, selected at random, over the Central United States. During the summer, when the 0° C level was located on average at about 15 000 ft, and cloud base at about 5000 ft, no precipitation was detected in those clouds with tops below 12 000 ft and warmer than +6° C, and only two of the twenty-nine clouds lying entirely below the 0° C level produced radar echoes. Of the clouds reaching the –12° C level (23 000 ft), about 30 per cent produced echoes, the probability of

shower production increasing rather sharply to over 80 per cent when the cloud surpassed the -20°C level at about 27 000 ft. These statistics contrast rather strongly with those for the New Mexico clouds, and suggest that shower formation proceeds more easily in the humid Central United States than in the arid south-west.

Because the majority of the precipitating clouds was appreciably supercooled, one might infer that the ice-crystal process plays an important role in the release of showers in the Central United States. However, Battan and Braham found in these clouds appreciable numbers of water drops exceeding $100\text{ }\mu\text{m}$ in diameter, which must have been produced by condensation and coalescence. Further evidence in favour of the coalescence process was published earlier by Battan (1953) from an analysis of the ground-radar records of the Thunderstorm Project obtained in Ohio during the summer of 1947. Although the tops of the visual clouds invariably reached the -20°C level, and often a great deal higher, in about 60 per cent of the 123 cases the whole of the initial radar echo was located at temperatures above 0°C . The warmest initial echo had its base at 20°C and its top at 14°C , the corresponding temperatures for the coldest echo being 10°C and -6°C respectively. There was a definite tendency for the colder echoes to be associated with lower freezing levels, so that the cloud thickness appeared to be a more important parameter than the temperature difference in determining the onset of precipitation. Battan is of the opinion that, in most cases, the initial radar echo was produced as the result of droplet growth by coalescence, starting, perhaps, on large hygroscopic nuclei. From the appearance and subsequent development of the echoes, and the fact that updraughts of several metres per second existed in these clouds, he argues plausibly against the possibility of snowflakes formed higher up falling, melting, and growing by accretion to produce the first radar echo below the 0°C level.

One obvious difference between the clouds studied by Battan and Braham and those observed in New Mexico is that with the 0°C level at about 15 000 ft in both regions, the cloud base was located at about 5000 ft in the former case and at about 10 000 ft in the latter; thus the clouds in the Central United States had an extra 5000 ft of non-supercooled cloud in which to accommodate drops growing by coalescence. On the other hand, Battan (1963) found that while many of the initial radar echoes from convective clouds in Arizona appeared at temperatures above 0°C , the altitudes of the centres of the echoes were positively correlated with the altitudes of the cloud bases in such a way

that, the lower the cloud base, the smaller the vertical separation of the cloud base and the initial echo. He took this as evidence for the initiation of rain by the coalescence process on the argument that the higher the liquid-water concentration associated with lower and warmer cloud bases, the lower would be the region of early raindrop growth. However, it is most unlikely that the observed differences in behaviour between cumulus clouds occurring in different regions and under different meteorological and topographical conditions are to be interpreted in terms of microphysical processes alone. The dynamical and thermodynamical processes that will govern such factors as the lifetimes, dimensions, and vertical motions of the clouds; their degree of mixing with the environment and hence their liquid-water content and buoyancy; the temperature, humidity, and wind structure of the atmosphere as manifested by such factors as stability and wind shear, must surely be of fundamental importance.

Indeed there is evidence for this in Battan's (1965) later radar and photogrammetric studies of clouds in Arizona. Finding that the correlation between the percentage of clouds producing radar echoes and the cloud-summit temperature was virtually the same on days of both heavy and light rain, he concluded that the duration and intensity of the rain reaching the ground were determined mainly by the size, strength, and duration of the updraughts.

Turning now to observations made in Australia, Bowen and his colleagues, working in Sydney, have clearly shown that rain may be released by either the ice-crystal process, or by coalescence from non-supercooled clouds, in this warm temperate region. Day (1953) classified the radar echoes observed throughout the year 1950 at Sydney into three categories: those showing melting bands indicative of the Bergeron process (30 cases), non-freezing showers (17 cases), and complex echoes such as are produced by cold-frontal showers, thunderstorms, and ill-defined cloud systems (24 cases). Day found that the Bergeron-type rain was associated predominantly with low-pressure systems and unstable air, and the non-freezing rain with high pressures and stable air when the clouds were unable to surpass the 0°C level.

A well-instrumented aircraft fitted with a radar set has been used with good effect by Bowen (1951), Smith (1950, 1951), and Styles and Campbell (1953), who give case histories of the precipitating clouds compiled from visual and radar observations. There were twelve occasions on which showers were observed to fall from clouds entirely below the 0°C level. All these clouds had base temperatures above

10°C , were (with one exception) more than 5000 ft thick, and were located either over the sea or near the coast where large salt nuclei were probably present in sufficient quantities to start the coalescence process.

Radar evidence for the production of showers by the coalescence process in England was obtained by Feteris and Mason (1956). On 13 days during the summer of 1955, 48 separate echoes were observed to originate below the -2°C level and 26 of these remained there during the whole life of the shower. These echoes, which were often associated with mixed formations of cumulus and stratocumulus cloud, sometimes extended above the 0°C level at their maximum state of development, but surpassed the -8°C level on only one occasion. On 9 of the 13 days, vertical development of the clouds was checked by the existence of stable layers usually below the -5°C level. The authors point out the limitations of using radar alone to study the initial stages of precipitation development; in particular, the stage at which a radar echo will first appear will obviously depend upon the range and reflectivity of the cloud and upon the performance characteristics of the particular radar. In order to obtain really conclusive information, the radar observations should be closely coordinated with measurements on the visual cloud.

6.3.2. *Observations on clouds in tropical and subtropical latitudes*

The paucity of well-documented information on the characteristics of the clouds that produce precipitation in tropical and subtropical regions has already been mentioned. However, during recent years, observations have slowly accumulated which show that heavy rain may fall from clouds which are entirely below the 0°C level, while the theoretical developments described in a later section suggest that conditions in the tropics may be more favourable for the coalescence process than elsewhere. There are therefore good reasons for believing that drop growth by coalescence may be predominant mechanism for releasing showers from tropical clouds.

Isolated reports of rain falling from non-accumulated clouds have been

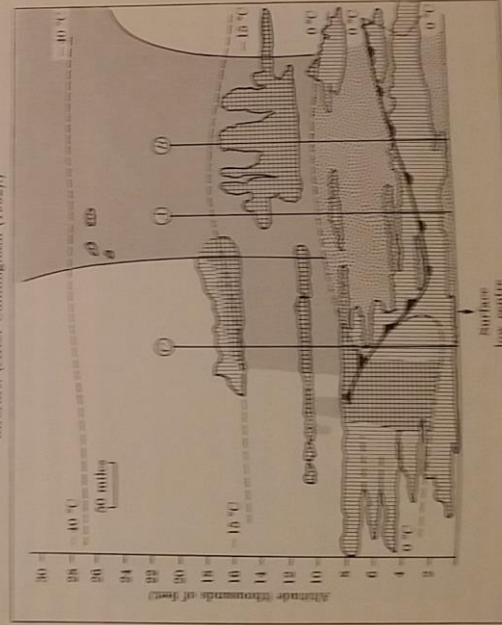


FIG. 6.3. Vertical cross-section, based on aircraft observations, through an extensive cyclonic layer aloft system. The flight path is that marked DD in Fig. 6.2. The alone drifting represents low crystals or snow, the vertical hatching water cloud, the cross-hatching areas of high supercooled water and snow, and the open dotting rain. (After Cunningham (1959)).

because the g
Layer clouds of thickness greater than a kilometre rarely occur well above the ground except in storm regions where the great thickness and extent of the layers, and persistent vertical motions of 10 cm s^{-1} or more, make the production of widespread rains by the ice-crystal process almost inevitable.

6.5. The release of precipitation from shower clouds

6.5.1. *Characteristics and structure of shower clouds*

Precipitation from shower clouds is generally of greater intensity and shorter duration than from layer clouds, the elements generally being

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of larger mass. The greater vertical depth of the clouds, their updraughts and higher concentrations of liquid water, all favour rapid growth of the precipitation elements by accretion. Precipitation from vigorous shower clouds and thunderstorms may reach the ground in the form of raindrops, pellets of soft hail, or as hailstones; snow showers are generally associated with weak or decaying clouds. Before discussing the growth of precipitation particles it is necessary first to consider the large-scale features of cumulus, and how they determine conditions that must be fulfilled before shower production can begin.

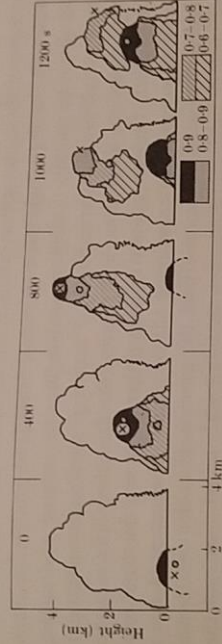


FIG. 6.11. Schematic representation of the structure of cumulus. The approximation of the liquid-water content to that in air rising from the bases *unmixed* is indicated for two successive rising 'bubbles' of cloud air. The successive positions of two particles, one of which grows into a precipitation element, are also shown. (From Ludlam (1952).)

The careful observation of cumulus and the experience of glider pilots in using the updraughts both in and beneath the clouds suggest that they are composed of rising bubbles of buoyant air, called thermals, which have a diameter of about 1 km. These thermals seem to have well-defined upper surfaces, but in constructing a model of the cloud structure, Scorer and Ludlam (1953) suppose them to have extensive wakes in which there is mixing with the outside air. At any moment a large cloud may contain a number of thermals, and for the rest, be composed of their wakes or the residues of earlier thermals, as suggested schematically in Fig. 6.11. Once a thermal reaches the periphery—the summit or the flanks—of the cloud, the mixing and wastage into a wake is accelerated; the rapid evaporation at the cloud borders chills the air there and destroys its buoyancy or even produces sinking, whereas in the interior of the cloud, the thermal is to some extent protected and is able to rise much further than if it ascended through clear air. A cumulus therefore has a characteristic pyramidal shape and viewed from a distance appears to have an unfolding motion, fresh

that must be fulfilled before shower production can begin.

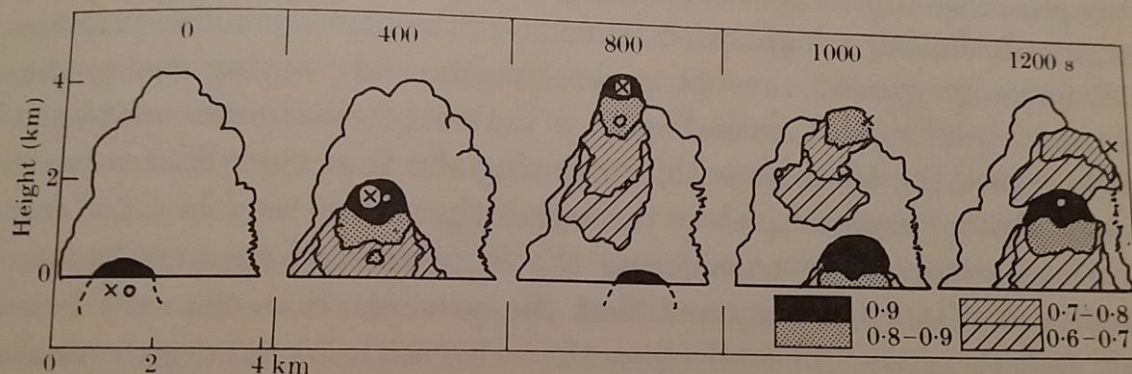


FIG. 6.11. Schematic representation of the structure of cumulus. The approximation of the liquid-water content to that in air rising from the bases *unmixed* is indicated for two successive rising 'bubbles' of cloud air. The successive positions of two particles, one of which grows into a precipitation element, are also shown. (From Ludlam (1952).)

cloud masses continually emerging from the interior to form the summit, and then sinking aside and evaporating. The else of the cloud depends not only on the properties of the environment, for example temperature, humidity, and stability, but upon the rate of supply of warm thermal air into the cloud base, which is determined by the rate of heating of the surface air and other factors which are, as yet, hardly understood.

Fig. 6.11 is taken from a paper by Ludlum (1952) and indicates hypothetically the approximation of the liquid-water content to the value corresponding to ascent from the cloud base without mixing. The strength of the updraught is similarly distributed, and it is clear that an aircraft making a high-speed traverse of the cloud would experience turbulence in passing through several regions of varying updraught, whereas a glider pilot ascending in a region of maximum updraught would regard the air as smooth, for during the time the glider takes to sink at about 1 m/s completely through a thermal it may be lifted into the cloud summits. In considering the formation of showers in large cumulus, Ludlum has assumed that the precipitation elements begin their growth in the thermal cores that rise without mixing from the cloud base to near its top. (Recent measurements indicate that liquid-water contents very close to the theoretical maximum values are sometimes encountered inside large cumulus clouds.) In other words, we assume the most favourable conditions for shower formation, so that the effect of any dilution of the cloudy air must be to increase our subsequent estimate of the size of a cloud which is capable of producing a shower.

In Fig. 6.11 are marked the positions within a thermal of two particles inside the cloud that begin to grow into precipitation elements; these may be regarded as large condensation nuclei that grow within the cloud by coalescence, or as ice nuclei that grow by accretion into ice pellets in the supercooled part of the cloud. One of these particles grows faster, being either larger initially or active as an ice nucleus at a higher temperature, and is shown as settling out of the air that carried it up into the cloud; it remains in the interior of the cloud, enters a newly rising thermal, and completes its growth as a raindrop or hailstone. Meanwhile the other fails to acquire a substantial fall speed before being carried to the outside of the cloud, where it is quickly evaporated together with the neighbouring cloud droplets.

In order to become a precipitation element, a particle must reach some rather critical size during the time taken for cloud air to rise from the base to the summit, such that it shall have settled some hundreds

of metres or more relative to the air, gained a fall speed about equal to that of the updraughts near the cloud top (c. 1 m s^{-1}), and have the capacity for falling with insignificant shrinking through several hundreds of metres of unsaturated air—and hence the opportunity to survive even if caught in the general evaporation of a cloud turret—and so resume its growth by settling into a newly rising mass of air within the cloud bulk. Ludlam (1951*a*, 1952) shows that these conditions are simultaneously satisfied when the particle has a radius of about $150 \mu\text{m}$ in the case of a water droplet, and a radius of either 550 or $300 \mu\text{m}$ in the case of an ice particle, depending on whether a mean density of 0.1 or 0.3 g cm^{-3} is assumed for the ice. Whether or not particles can attain this critical size, and hence the probability of shower formation, will depend on the depth of cloud traversed, in turn determined by the speed of the updraught, and the liquid-water concentration which is largely governed by the temperature of the cloud base and the degree of mixing of the cloudy air with the drier environment. If the entire cloud is small, its life may be only a little longer than the time taken for particles to reach the critical size, so that the particles are prevented from growing into precipitation elements; but if the cloud is large, a shower is almost certain to develop, although a strong wind shear may intervene both by reducing the life of the cloud and by causing the large particles settling out of the cloud tops to fall into clear air rather than into succeeding thermals where they may continue their growth.

We shall now discuss various mechanisms by which precipitation may be released from shower clouds and present the results of some calculations on the growth of precipitation elements.

6.5.2. Production of showers by coalescence of cloud droplets

The fundamental equations for the growth rate of a drop by coalescence with smaller cloud droplets may be written as follows. The rate of mass increase of a drop of radius R , falling with a velocity V through a cloud of liquid-water content w , composed of smaller droplets having terminal velocity v , is given by

$$dm/dt = 4\pi R^2 \rho_L \frac{dR}{dt} = E' \pi R^2 w (V - v), \quad (6.6)$$

$$\text{i.e.} \quad dR/dt = E' w (V - v) / 4 \rho_L, \quad (6.7)$$

where E' is the collection cross-section of the drop and ρ_L its density. Its rate of vertical displacement above a fixed level is

$$dz/dt = U - V, \quad (6.8)$$

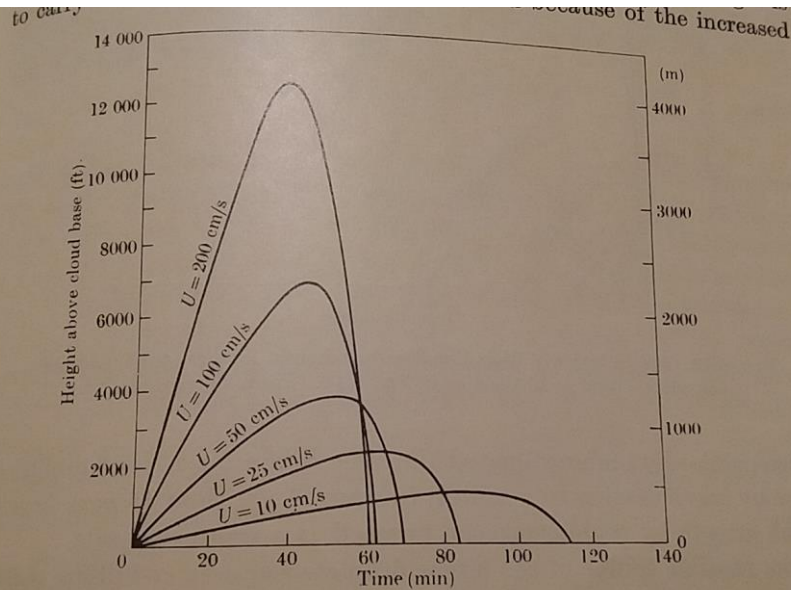


FIG. 6.12. The trajectories of drops which grow by coalescence in clouds having updraughts of 10 to 200 cm s^{-1} . Water content = 1 g m^{-3} ; cloud droplet radius $10 \mu\text{m}$; percentage supersaturations are one-thousandth of the updraught velocities. (From Bowen (1950).)

where U is the updraught velocity. Thus the increase of radius with height is given by the equation

$$dR/dz = E'w(V-v)/4\rho_L(U-V) \simeq E'wV/4\rho_L(U-V). \quad (6.9)$$

If U is considered to be constant with height, and E' and V to be functions only of R , eqn (6.9) may be integrated to give the radius of the drop at level z , viz.

$$4\rho_L \int_{R_0}^R \frac{(U-V) dR}{E'V} = \int_{z_0}^z w dz, \quad (6.10)$$

where R_0 is the initial radius of the drop at the starting level z_0 . Evaluation of the integrals in eqn (6.10) requires a knowledge of the distribution of liquid-water content with height, and of the collection cross-section E' and the terminal velocity V as functions of the drop radius R (see Appendices A and B).

Langmuir (1948) tabulated values of

$$4\rho_L \int_{R_0}^R dR/E' \quad \text{and} \quad 4\rho_L \int_{R_0}^R dR/E'V$$

for various values of R_0 and R , and used them to compute the growth of water drops by coalescence in clouds in which the water content was constant with height and the updraught was zero. He calculated that a drop of initial radius $40 \mu\text{m}$ would have to fall 13 km through a cloud of water content 1 g m^{-3} and composed of droplets of $8 \mu\text{m}$ radius in order to attain a radius of 3 mm , the time taken being 64 min ; for other values of water content, the distances and times would be inversely proportional to w . The distances calculated by Langmuir are considerably reduced if one assumes the presence of an appreciable updraught in the cloud. In this case, the incipient raindrop may be carried up in the rising air, growing by coalescence with the smaller cloud droplets until it reaches a level at which its rate of fall exceeds the speed of the updraught, when it will return towards the cloud base, growing as it descends.

Calculations on the growth of drops by coalescence in clouds with constant updraught velocity and liquid-water content were made by Bowen (1950), using values of E' computed by Langmuir. A drop, originating by the chance coalescence of two cloud droplets near the cloud base, was assumed to grow by condensation at a supersaturation which was a linear function of the updraught velocity until it reached a

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radius of $15\text{ }\mu\text{m}$ at which, according to Langmuir, it attained a finite collection efficiency for the cloud droplets of radius $10\text{ }\mu\text{m}$. Assuming a uniform concentration of liquid water of 1 g m^{-3} , Bowen computed a number of height-time curves for various values of the updraught velocity (Fig. 6.12) and a corresponding series of curves of height against drop diameter (Fig. 6.13). The effect of a stronger updraught is to carry the drop higher into the cloud, and because of the increased

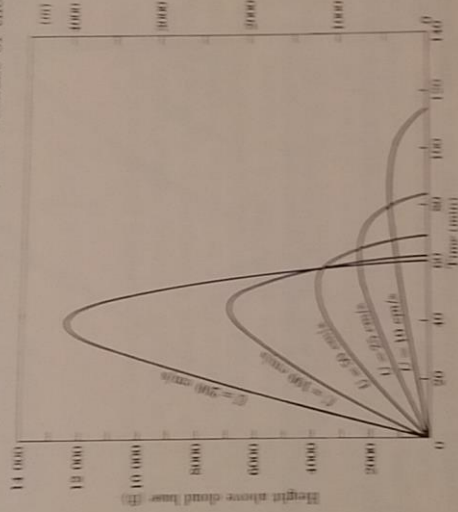


FIG. 6.12. The trajectories of drops which grow by condensation in clouds having updraughts of 10 to 300 cm s^{-1} . Water content = 1 g m^{-3} , cloud droplet radius $10\text{ }\mu\text{m}$; percentage supersaturations are one-thirtieth of the updraught velocities. (From Bowen (1959).)

double fall path, to produce a larger drop at the cloud base. That Fig. 6.12 appears to suggest that a stronger updraught would produce a larger drop in a shorter time, is a consequence of Bowen's assumption that a stronger updraught is associated with a higher supersaturation and more rapid growth of the drop in the early stages. Bowen's calculations also indicate that a threefold increase in the water content would cause an increase of only 33 per cent in the final drop size, because although the growth rate increases with larger values of w , the length of the path over which growth takes place is reduced. It can be seen, therefore, that the final size of the drops is relatively insensitive to changes in liquid-water content. However, mainly because Langmuir's values of collision efficiency are in error for small droplet sizes,

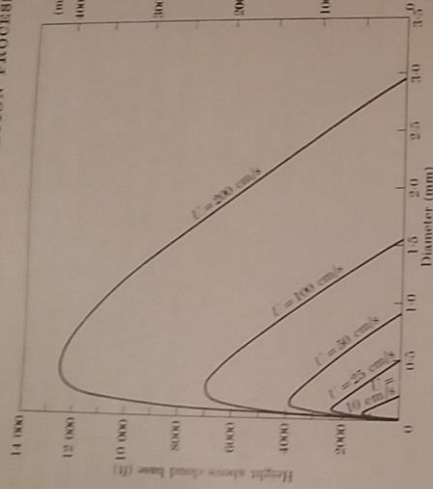


FIG. 6.13. The variation of drop diameter with height for a range of updraught velocities. Other conditions as in Fig. 6.12. (From Bowen, loc. cit.)

Bowen seriously underestimated the time taken for a cloud droplet to grow by condensation to a radius, (nearer $25\text{ }\mu\text{m}$ than $15\text{ }\mu\text{m}$), beyond which growth by coalescence can proceed at a significant rate.

One possible source of the larger droplets required to initiate coalescence in shower clouds is the giant hygroscopic salt nuclei found over the sea, with masses exceeding 10^{-9} g and in concentrations of order $1/\text{litre}$. They will leave the sea surface as spray droplets of radius greater than $20\text{ }\mu\text{m}$, and in convective weather may be carried up into the cloud without appreciable evaporation. These particles may, therefore, be important for shower production in unstable maritime air in warm weather. As the air moves inland, they may disappear by sedimentation and shrink by evaporation towards their equilibrium radii appropriate to the prevailing relative humidity. Thus a salt nucleus of mass 10^{-9} g might shrink to a radius of about $8\text{ }\mu\text{m}$ in an atmosphere of 60 per cent relative humidity, but on re-entering a cloud, would take only 3–4 min to grow to a radius of $25\text{ }\mu\text{m}$; a nucleus of mass 10^{-10} g would, in similar circumstances, take about 30 min to reach this size.

If the drop enters the cloud with a radius R_0 and leaves it with a radius R , U is constant with height, and w is considered independent of

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time (i.e. we ignore depletion of the liquid water by the growing drops), eqn (6.10) becomes

$$4\rho_l \int_{R_0}^R \frac{(U - V)}{R^2 V} dR = \int_{z_0}^z w dz = 0, \quad (6.11)$$

Ludlum (1951a) argued that, except in the lowest part of the cloud where, in any case, growth by coalescence is very slow, V may be considered as a function of z only, or even to be constant with height. Eqn (6.11) then becomes

$$R - R_0 = U \int_{R_0}^R \frac{dR}{V}, \quad (6.12)$$

showing that the radius of a raindrop falling out of a cloud is a function only of the size at which it originally entered the cloud and the speed of the updraught. Values of R computed by Ludlum for various values of U and R_0 are shown in Fig. 6.14. The size of the raindrops increases with the speed of the updraught, until at a speed of between 3 and 4 m s⁻¹, they reach maximum stable size ($R \approx 2.5$ mm). If the critical speed is exceeded, drop breaking occurs within the cloud. It is interesting to note that the large raindrops may develop in updraughts having speeds only a third or a quarter of their fall speed. Fig. 6.14 shows that for a given value of U , the largest raindrops result from

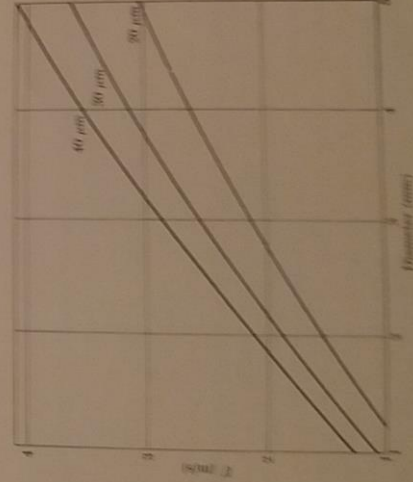


FIG. 6.14. Diameters of raindrops grown from droplets of initial radius 90, 80, and 40 μ m as a function of the steady updraught U . (From Ludlum (1951a))

smaller initial drops, as these are carried up farther into the cloud and so have a longer fall path. This does not necessarily imply that smaller droplets are more efficient in releasing rain, for although their growth is potentially greater, it is accomplished over only a longer period that may extend beyond the life of the cloud.

So far the water content w has been eliminated from consideration, but we must introduce this if we wish to know whether raindrops can actually be produced. A low value of w might mean that insufficient time is available, or that the growing droplet would be lifted to a great height at which it might freeze, and therefore merely contribute to the ice phase; similarly the cloud might not be deep enough to allow unrestricted growth and the particle might be carried out in a thermal at the top of the cloud and not be large enough to survive evaporation before falling back into a newly rising cloud mass. As explained in § 6.5.1, Ludlam takes as a criterion for shower production that air carried to the cloud summits shall contain drops of radius $R > 150 \mu\text{m}$. Drops of this size, if they settle into a freshly rising cloud mass at the top of the cloud (where updraughts will generally not exceed 1 m s^{-1}), will still recede from the summit level and continue to grow. Also, drops of $R > 150 \mu\text{m}$, if carried 300 m above the cloud top into air of relative humidity 95 per cent, will evaporate very little while falling back into the cloud, whereas appreciably smaller ones ($R < 120 \mu\text{m}$) will evaporate almost completely.

Thus, the necessary condition for shower formation is the growth of some drops to $150 \mu\text{m}$ radius during the time taken for air to rise from cloud base to summit. From eqn (6.10) we have

$$U \int_{R_0}^{150\mu\text{m}} \frac{dR}{E'V} - \int_{R_0}^{150\mu\text{m}} \frac{dR}{E'} = \frac{1}{4\rho_L} \int_{z_0}^{z_b} w dz, \quad (6.12a)$$

where z_b is now about 300 m less than the critical summit level that must be reached if a shower is to develop. Taking a mean value of $E' = 0.85$, Ludlam calculated the critical minimum depth z_b of a cloud as a function of U for initial droplet radii $R_0 = 20, 30, 40 \mu\text{m}$ in cloud of base temperatures -5°C and 20°C , as shown in Fig. 6.15. It is seen that the critical depth of cloud necessary for shower formation increases considerably with the speed of the updraught and is also somewhat larger for colder clouds and smaller values of R_0 . According to these computations, if droplets of initial radii $20\text{--}30 \mu\text{m}$ are present, clouds of only 1.5 to 4 km depth are required to produce showers by the

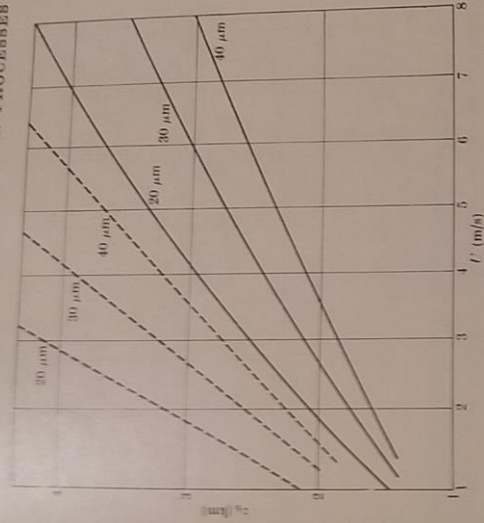


FIG. 6.15. The variation of the minimum cloud depth z_b for shower development as a function of the updraught U , for initial droplet radii of 20–40 μm in clouds with base temperatures -5°C (pecked lines) and 20°C . (From Ludlam, 1960, *op. cit.*)

coalescence mechanism. The time required for the growth of a raindrop is found by adding a few minutes to the value obtained by dividing the critical depth z_b by the velocity of the updraught, and varies from about half an hour with $U = 1 \text{ m s}^{-1}$ to about 20 min in the stronger updraughts. In order for a considerable shower to occur, the life of the entire cloud must exceed these periods by at least several minutes. Fig. 6.16 shows the variation with base temperature (which largely controls the liquid-water content—see Fig. 6.17) of the minimum cloud depth likely to be associated with showers when the updraughts are weak ($U = 1 \text{ m s}^{-1}$), and the critical depths for some other conditions shown on the right-hand side. If the cloud-base temperature exceeds 8°C , showers may fall from clouds whose summits do not reach the 0°C isotherm. If the base temperature approaches 20°C , the shower-cloud tops are not supercooled even with updraughts of 8 m s^{-1} . On the other hand, in the colder clouds, moderate updraughts may lift the growing drops to temperatures below 0°C where they may freeze, particularly if the temperature is below -20°C .

radius in clouds of the observed depth and, indeed, fibrillation was not often observed. Nevertheless, small cumulus sometimes produced *virga* and light showers even when the cloud depth was as little as 900 m and the summit temperature as high as $+3^{\circ}\text{C}$. On such occasions, the cloud summits entered a very stable layer or an inversion and spread out to form expanding shelves of strato cumulus cumulonimbus; the evaporation of the cloud summits was then very slow, and the growth of the precipitation elements probably continued over a long period within a succession of thermals.

6.5.3. *Raindrop multiplication by break-up of large drops*

Fig. 6.14 shows that raindrops of radius 2.5 mm may be produced if the updraught exceeds 3 or 4 m s^{-1} . Falling drops with radii greater than 1 mm suffer a distortion in shape, which becomes ever more pronounced until a stage is reached at which any further increase in mass results in disruption of the drops. Experiments, described in Appendix B, in which water drops are suspended in a vertical air-stream, indicate that break-up generally occurs when the drop diameter exceeds 6 mm, thus accounting for the fact that drops larger than this rarely occur in rain. Although drops of up to 8 mm diameter may remain stable for periods of many minutes in a non-turbulent, uniform air-stream, drops of diameter greater than 5 mm are extremely sensitive to the degree of micro-turbulence and to any sudden changes in the air velocity, and in a non-laminar current, will break up within a very short time. Break-up may also follow the collision of two raindrops, the stability of the resultant mass depending on the size, shape, and phase of oscillation of the two drops at the moment of impact. The number and size of the fragments produced during disruption also depend very much on these last-mentioned factors and on the velocity fluctuations of the air stream, but, under the conditions which might be expected to exist in a shower cloud, usually several drops of about 1 mm radius and much larger numbers of much smaller ones are produced.

Now each of the large fragments may continue to grow by coalescence and may reach a size at which it, in turn, breaks up. In the case of a very deep cloud with high water content, it is conceivable that the process might repeat itself a number of times, so that the number of raindrops is rapidly increased by a kind of chain reaction. Eventually, however, their accumulation will overcome the buoyancy of the air which sustains the upcurrent, so that they will be released as a heavy

breaking is unlikely to occur in such cold clouds but will be favoured in warm clouds of high water content.

Some unpublished calculations by the author show that in a cloud with base at 10°C and 900 mb and sustained by a uniform updraught of 7 m s^{-1} , a droplet of $40\text{ }\mu\text{m}$ entering the cloud base would be carried up to a height of 3.5 km above cloud base (temperature -10°C) before commencing to fall through the updraught, and would attain break-up size (radius 2.5 mm), at the 3.2 km level after a total period of 12 min. If the disrupting drop is assumed to produce fragments of 1 mm radius, these, in turn, will grow to break-up size about 3 min later, after a fall of about 400 m. Successive disruptions will take progressively longer because of the lower water contents at lower levels, so that in the above cloud model, the maximum possible number would be six. However, these calculations are made on the assumption that the cloud contains the adiabatic liquid-water content and that this remains constant during growth of the drops. In practice, it is likely to be progressively depleted, and in consequence the chain reaction will gradually run down. Thus even in the centre of a large, warm cumulonimbus, it is doubtful whether a drop may undergo more than three successive breakings; these would require more than 10 min, by which time the original number of raindrops may be increased a thousandfold, building up in a localized region a concentration of rain-water sufficient to destroy the updraught and release a heavy shower.

6.5.4. *The release of showers by the growth of ice particles*

In a further paper, Ludlam (1952) made analogous calculations to those described in § 6.5.2 for the case when showers are initiated by the growth of ice particles.

The newly-formed ice particles grow at first by deposition of water vapour but, at temperatures between -10 and -20°C , soon attain a size and fall speed at which their growth continues predominantly by the capture and freezing of cloud droplets. Thus at -14°C , when the difference between the saturation vapour densities of water and ice is a maximum, a plate-like crystal of thickness $10\text{ }\mu\text{m}$ will grow in a water-saturated environment to a radius of $50\text{ }\mu\text{m}$ in 100 s, while a droplet growing on a giant salt nucleus of 10^{-9} g will achieve a radius of only $20\text{ }\mu\text{m}$ in this time. Subsequent growth of the ice particle is partly by sublimation, and partly by the accretion of supercooled droplets that freeze on contact to form a loosely-packed aggregate with air spaces in between. The density of this rimed particle, ρ_i , may therefore be quite

more recent and reliable computations of collection efficiency, and cloud models in which the updraught and liquid-water content vary with height in a realistic manner.

6.6. Hail

6.6.1. *Occurrence of hail*

Hailstorms occur most frequently in the continental interiors of middle latitudes and diminish towards the poles and the equator and over the sea. In cold climates, the clouds are not sufficiently vigorous, neither do they contain large enough concentrations of cloud water to produce large hailstones. Over the oceans, in the absence of intense surface heating, the cumulonimbus do not usually become organized into the large cloud systems that are usually associated with large hail. In the tropics, the strong horizontal temperature gradients and large vertical wind shears, which favour the development of damaging hailstorms (see p. 359), do not usually extend into the high troposphere.

Although the incidence of hail, particularly of large stones, is usually associated with thunderstorms, small hail may fall from clouds that do not reach thunderstorm proportions. Beckwith (1960), from an analysis of about 300 hailstorms occurring on 225 hail-days over the 10-year period 1949–58 in the Denver area of Colorado, concluded that hail fall on only about 1 in 8 of the days on which thunderstorms were reported by a given station, but that the numbers of hail incidents reported by the network of forty stations was four times that reported by any one single station. But none of these figures may be very reliable in indicating the proportion of thunderstorms that contain hail at some stage of their development, and Beckwith expresses the opinion of his forecasters and pilots that 'nearly every thunderstorm developing off the Rocky Mountains contains hail in some stage of its development.'

Large hail usually falls from only rather localized regions of the storm and for a brief period. The areas of severe hail damage on the ground may vary in width from a few yards to several miles, commonly 1 mile. The duration of a hail fall may vary from as little as 10 s to as much as 30–40 min, a median value being, perhaps, 3–5 min.

6.6.2. *Classification of hail*

The term 'hail' is applied to a variety of solid hydrometeors that fall from supercooled clouds. The International Commission for Ice and Snow recommended, in 1956, the following definitions which broadly categorize the various forms of such precipitation.

Snow pellets (soft hail, graupel). These are white, opaque, rounded or conical pellets of diameter up to about 6 mm. They are composed largely of small cloud droplets individually frozen together, have a low density, and are readily crushed. They may break up on striking a hard surface.

Ice pellets (small hail, grains of ice). These are transparent or translucent pellets of ice, spherical, spheroidal, conical, or irregular in shape, and having diameters of a few millimetres. There are two main types: (a), frozen raindrops or largely melted and refrozen snowflakes, the freezing process usually taking place near the ground and (b), pellets of snow encased in a thin layer of ice which has formed by the freezing of either of droplets intercepted by the pellets or of water resulting from the partial melting of the pellets.

Hail. Small balls or pieces of ice (hailstones) upwards of 5 mm in diameter. A moderately severe storm may produce stones a few centimetres in diameter, while a very severe storm may give rise to stones with maximum diameters of 10 cm or more. Hailstones are composed almost entirely of transparent ice, or of alternate layers of clear and opaque ice.

This classification could usefully be revised and extended in the light of the recent researches described in the following pages.

6.6.3. *The size and shape of hailstones*

Hailstones exhibit a wide diversity of shape—spherical, spheroidal, conical, discoidal, and irregular forms being found, sometimes several of these together in one storm. The roughly spherical form is perhaps the most common when the hail is small, but most large hailstones are not spherical. According to Macklin (1963), 60–70 per cent of the stones from a severe storm in SE. England were oblate spheroids and only a few per cent were spherical. Browning and Beimers (1967) report that the oblateness of large hailstones tends to increase during growth. Moreover, a further increase may be produced by preferential melting at the two ends of the minor axis during descent to the ground and, in extreme cases, this leads to the development of apple-shaped stones. The surfaces of the stones may be smoothly curved, or irregular with protuberances and spikes.

There have been few systematic observations of hailstone sizes; observers tend to report the maximum and mean sizes, but not the smaller sizes which, in any case, may melt on their journey to the ground, so that we have very little reliable information on the size

distribution of hailstones from individual storms. The size distributions of the pellets in three falls of soft hail on Wolschlojoh have been reported by List (1938). They were mostly conical pellets with apex angles ranging from 70 to 80°. Their diameters ranged from 0.5 to 5.5 mm, the most frequent diameter being 1.5 mm in two of the falls, and 2.5 mm in the third. Many seemed to originate from an ice crystal that could be identified in the apex of the particle and then grew by the collection and rapid freezing of supercooled cloud droplets. List also gives size distribution curves for two falls of *small hail*. Here the particle diameters range from 2.5 to 7.5 mm, the most frequent diameter being 4 mm for conical and irregular pellets and 6.5 mm for spherical pellets. The frequency distribution of the sizes of the *largest* hailstones observed in storms in the Denver area during 1949–55 is given by Beekwith (1956) as follows:

Size of largest hailstones	Number of onsets	Percentage
Grain ($< \frac{1}{4}$ in)	10	1.4
Current ($\frac{1}{4}$ in)	122	16.4
Pea ($\frac{3}{8}$ in)	282	45.0
Grape ($\frac{1}{2}$ in)	149	23.5
Walnut (1–1½ in)	38	6.0
Golf ball (1½–2 in)	26	4.1
Tennis ball (2½–3 in)	4	0.6

Hailstones of at least walnut size fell on at least 2 days per year—once for about every sixty thunderstorms.

The largest hailstones ever reported from the United States fell in Nebraska, the largest being 5.4 in (13.8 cm) in diameter and weighing 1½ lb.

A complete spectrum of the sizes of hailstones, ranging from 0.7 to 8.5 cm in diameter, that fell from a particular severe storm has been published by Ludlam and Macklin (1959), to which Atlas and Ludlam (1961) fitted the formulae $N(R) dR = N_0 \exp(-\Lambda R) dR$ for the spatial concentration, N , of the stones in the cloud, where $N_0 = 8 \times 10^{-3} \text{ cm}^{-4}$ and $\Lambda = 4.54 \text{ cm}^{-1}$ are constants.

An analysis by Carte (1963) of over 1100 reports of hail, made during 56 days of 1962–3 in Pretoria, South Africa, revealed that the largest stones reported were pea or grape size ($d < 1 \text{ cm}$) on 56 per cent of occasions, and golf-ball size or larger ($d > 3 \text{ cm}$) on 3 per cent of occasions. More recent analyses by Carte (1966), and Carte and Kidder (1966) of the reports of about 800 voluntary observers from an area of 1000 miles², where hail fell on 60–90 days per year, revealed that

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stones of $d > 3$ cm fall on < 5 per cent of occasions, and that these were often composed of clear ice having a knobby structure although smooth and opaque stones sometimes fell at the same time. Although some large stones consisted of soft slushy ice, they were more frequently hard, or had hard shells that may have formed as the result of freezing during storage. On the other hand, hail of $d < 1$ cm was frequently soft and composed of rime ice into which melted or congealed water had percolated. Classification of 3514 hailstones into five categories revealed that spheroidal stones outnumbered all other shapes in all size groups below 7-cm diameter, but both spheroidal and conical shapes were less dominant among the bigger stones, where ellipsoids, prolate spheroids, and irregular shapes were well represented. Disc- and lens-shaped stones were always rare, but about one-quarter of all the stones had a flattened profile.

6.6.4. The structure of hail

6.6.4.1. Types of ice structure

The internal structure of a hailstone may best be studied by examining thin sections of the stone under the microscope with transmitted and polarized light. Thin slices, preferably 0.3–0.4 mm in thickness, may be prepared either by cutting with a special horizontal saw described by List (1961) or by melting down a stone to the required thickness between two warm metal plates. In this way, one can study the size and arrangement of the individual ice crystals that compose the stone, the size and distribution of the air bubbles in the ice, and the location of the growth centres. The orientation of the larger crystals can also be determined from the orientation of either etch pits (Painter and Schaefer (1960), Koenig (1962), Aufdermaur, List, Moyes, and de Quervain (1963)), or epitaxial deposits of ice crystals (Hallett 1964), formed on their surfaces. There is a danger, however, that the storage, cutting and polishing of the stones may cause some re-crystallization and structural changes.

Within the complex structure of hail particles it is possible to distinguish three kinds of ice deposit:

(a) *Porous (rime) ice* is formed when the deposited water freezes rapidly, mainly as individual droplets, and produces a rimed structure. The air spaces between the frozen droplets give the ice a low density, perhaps as low as 0.1 g cm^{-3} in some cases. The frozen droplets contain large numbers of tiny air bubbles which, because of their high scattering power for light, give the ice an opaque, white appearance. Soft hail

pellets are largely composed of low-density rime ice and this is also found in the cores of some hailstones.

(b) *Compact ice* having density close to 0.9 g cm^{-3} is formed when the droplets have time to spread over the surface and form a continuous film before freezing. This happens when the transfer of heat between accreted water and the environment is just sufficiently rapid to allow all the division at 0°C . Since only rather small concentrations of air bubbles are released during freezing under these conditions, the ice is usually clear. Compact ice may also be formed on colder dry surfaces if the droplets impact at high speeds and spread before freezing. In these circumstances the ice may be opaque.

(c) *Spongy ice* is produced when the heat exchange between the hailstone and the environment is not sufficiently rapid to allow all of the deposited water to freeze. For a stone of given size and shape falling through a specified environment, there exists a critical value of the concentration of liquid water above which the surface becomes wet. Only a fraction of the collected water freezes immediately to produce a skeletal framework of ice that retains the unfrozen water, the whole mixture being maintained at 0°C . This wet coating of 'spongy' ice, of density $0.9\text{--}1.0 \text{ g cm}^{-3}$, is often quite transparent, but sometimes contains sufficient concentrations of small air bubbles to give it a milky appearance. In this wet condition, the hailstone may also collect ice crystals and snowflakes and grow rapidly in size.

6.6.4.2. The density, crystal structure, and air content of accreted ice

The conditions under which these different structural forms of ice are produced have been elucidated in the laboratory experiments of List (1959a, 1960) and Macklin (1961, 1962). Macklin examined the ice deposited on cylinders under a wide range of conditions: air temperatures from -5 to -30°C , air speeds from $2\frac{1}{2}$ to 12 m s^{-1} , cloud-water concentrations from 1 to 7 g m^{-3} , cloud droplets of mean-volume radii $11\text{--}32 \mu\text{m}$, and cylinder diameters from 1 to 14 mm . In the regime corresponding to the linear part of the graph in Fig. 6.19, Macklin found that the density of the rime deposit fitted the empirical formula

$$\rho_1 = 0.11(-rv_0/T_0)^{0.70},$$

where r is the mean-volume droplet radius in microns, v_0 the impact velocity of the droplets in m s^{-1} , and $T_0^\circ \text{C}$ the surface temperature of

to 17.3 g) from a single storm ranged from 0.875 to 0.912 and supported the high values found by other workers. Over 90 per cent of these stones had transparent centres, the diameters of which ranged from 1.5 mm to 1 cm. They were probably frozen raindrops that had continued to grow in the wet stage.

The specific gravities of 16 representative stones, with masses ranging from 7 to 50 g, from a violent South African storm, are quoted by Mossop and Kidder (1961) as lying between 0.87 and 0.90, with an average of 0.89.

It is apparent that the densities of hailstones are within about 5 per cent of that of clear ice. This is to be expected in view of the high impact speeds and surface temperatures associated with their growth.

6.6.6. *The aerodynamics of hailstones*

The aerodynamic resistance, or drag force, F , exerted by a bluff body in an air-stream of velocity V is given by

$$F = \frac{1}{2}AC_D\rho_a V^2, \quad (6.15)$$

where A is the projected cross-sectional area of the body normal to the flow, ρ_a the air density, and C_D is the drag coefficient. A body in free fall attains its terminal velocity when the drag force is balanced by the gravitational minus the buoyancy forces. For *smooth spheres*, C_D remains constant at about 0.45 over the Reynolds number range 10^3 to 4×10^5 but, at this latter critical value, C_D decreases abruptly to about 0.1 because the whole of the boundary layer of the sphere becomes turbulent.

Measurements of the drag coefficients of natural hailstones, hailstone replicas, and objects resembling hailstones up to 6 cm in diameter, have been made by a number of workers. List's (1958b) measurements on wooden models in a wind tunnel suggested that the drag coefficient of a small conical hail pellet, falling with its apex pointing upwards, would be about 1.0. He also made measurements on four large hailstones, three ellipsoidal and one spherical, varying in mass from 18.6 to 32.4 g. When they were oriented with the shortest axis in the direction of the wind, (this is the stable position in free fall), the drag coefficients were about 0.65 compared with 0.45 for a smooth sphere. These results imply that the fall speeds of ellipsoidal stones and of spherical stones with a rough surface must be considerably lower than those of smooth spheres having the same mass.

List's measurements were confirmed in a general way by Macklin

and Ludlam (1961), who measured the fall speeds of seventeen large model hailstones made by freezing water in moulds. The stones were placed in a box, carried aloft on a balloon, tipped out at a suitable height, and their falling speeds measured with radar. The corresponding drag coefficients for smooth, spherical stones, of Re up to 1.4×10^6 , were close to 0.45, but oblate spheroids had values between 0.7 and 0.8 and much smaller falling speeds than spheres of the same mass. The drag coefficients of three real hailstones, of masses between 14.7 and 21.8 g, were measured in a wind tunnel, the values being 0.5-0.6 for spherical stones with $Re = 2.5-6 \times 10^4$, and about 0.7 for oblate spheroids orientated with their shortest axis along the wind direction.

Young and Browning (1967) investigated the effect of surface roughness on the drag coefficients of spheres in a wind tunnel, having a very low level of free-stream turbulence. While the drag coefficient of smooth spheres rose from 0.43 to 0.53 as the Reynolds number, Re , was increased from 4 to 20×10^4 , spheres with rough surfaces experienced a sharp drop in drag at critical values of Re ranging from 7 to 17×10^4 as the roughness parameter (ratio of height of roughness elements to sphere diameter) was increased from 1.3 to 13×10^{-3} . These results are consistent with those of Willis, Browning, and Atlas (1964), who made simultaneous measurements of the fall speeds and radar cross-sections of ice spheres of $d \approx 5$ cm falling in free air from about 20 000 ft (6 km). While they were dry, the spheres fell with slightly supercritical Reynolds numbers ($Re > 1.3 \times 10^6$) and with drag coefficients of only 0.24 to 0.30. The surface of one sphere, 5.1 cm in diameter, became wet during its fall, whereupon the Reynolds number fell to 0.98×10^6 and the drag coefficient increased to 0.56, probably as a result of the boundary layer undergoing a transition from turbulent to laminar flow when the first layer on the sphere melted.

Bailey and Macklin (1968a) have confirmed that the drag coefficients of roughly spherical artificial hailstones, 4-6 cm in diameter and with moderately rough surfaces, may drop quite sharply when Re exceeds 10^5 , but finds that this is not the case for larger stones with pronounced lobes, which have drag coefficients of 0.4 to 0.5 even when the Reynolds numbers exceed 3×10^5 . It seems, then, that large hailstones, unlike smooth spheres of similar size, do not enter the supercritical flow regime.

6.6.7. Theories of hailstone growth

The early theories of hail formation held that hailstones grew either by the aggregation of cloud elements at temperatures below $0^\circ C$, or by

deposition of water vapour directly into ice. When the multi-layered structure of relatively large stones was discovered, the formation of the opaque low-density layers was usually explained in terms of the collection of snow on the accretion and rapid freezing of supercooled droplets in cold regions of the cloud, while that of the transparent layers was attributed to the acquisition of a coating of liquid water in the non-freezing regions which could freeze only when the hailstone was again carried above the 0°C level. The occurrence of alternate transparent and opaque layers was then explained by its oscillation above and below the 0°C level under the influence of a pulsating updraught. A necessary consequence of this theory was that updraughts of the same magnitude as the terminal velocity of the largest formed hailstone should exist in the cloud in order to carry it up during the penultimate stage of its growth. The appearance of hailstones 5 cm in diameter would therefore imply updraughts of about 30 m s^{-1} .

An important advance in the theory was made by Schumann (1938), who produced the first really quantitative treatment of hailstone growth. He assumed a hailstone to grow as a spherical particle by sweeping up all the supercooled water droplets lying in its fall path. Ignoring the early stages of growth, he followed its history only after it had attained a diameter of 0.5 cm and calculated the ultimate radius in terms of the distance travelled, the liquid-water content w , and the updraught velocity U in the cloud. He used equations equivalent to eqns (6.6)–(6.9), integrated and written in the form

$$4\rho_1 \int_{R=0.25\text{ cm}}^R \text{d}R = \int_0^h w \text{d}h \quad (6.16)$$

or

$$\rho(4R-1) = wh, \quad (6.16)$$

where ρ_1 is the particle density and h the distance travelled by the particle relative to the air. Also,

$$4\rho_1 \text{d}R/\text{d}t = w \text{d}h/\text{d}t = wV, \quad (6.17)$$

where V is the terminal velocity of the hailstone.

Using the relation $V = 2000 (\rho_1 R/\delta)^{\frac{1}{2}}$, which is based on the calculations of Bilham and Relf (1937), and where δ is the specific gravity of air with respect to air at 0°C and 1000 mb pressure, Schumann used eqn (6.17) to calculate the distance $z = h - Ut$ travelled by the hailstone relative to a fixed level, viz.

$$z = \frac{\rho_1}{w}(4R-1) - \frac{U(\rho_1 \delta)^{\frac{1}{2}}}{1000w}(4R^{\frac{1}{2}}-2). \quad (6.18)$$

He considered, as an example, a particle starting at the -20°C level (pressure 420 mb) with radius 0.25 cm and followed its subsequent growth over a fall path of $z = 3$ km, the average density ρ , being 0.6 g cm^{-3} . The final radius of the hailstone as a function of the liquid-water content and the updraught velocity is shown in Fig. 6.26, from which we may infer that a hailstone of 2.5-cm radius could fall from a cloud of liquid-water content 6 g m^{-3} and an average updraught of only 15 m s^{-1} .

From these and other similar calculations, Schumann concluded that, for the growth of large hailstones, the clouds must extend to heights of

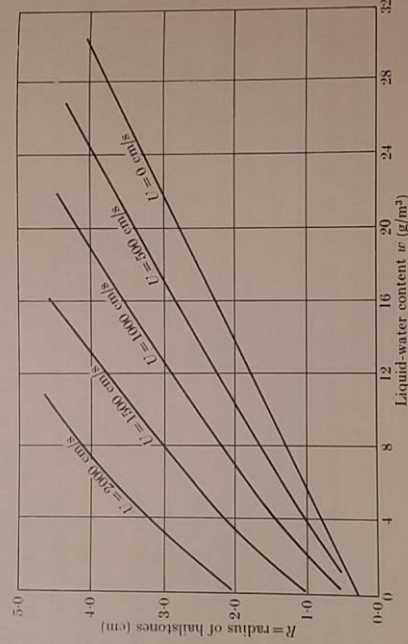


FIG. 6.26. The final radius attained by a hailstone of mean density 0.6 g cm^{-3} falling from a height of 7 km for different values of liquid-water content and updraught velocity. (From Schumann (1938).)

several kilometres, and that they must contain either very high concentrations of liquid water or very strong updraughts. However, it is noteworthy that his calculations indicate that clouds of moderate water content may produce hailstones exceeding 1 inch in diameter and that the updraught velocities need be only about one-half the terminal velocities of the stones. But one must be careful in drawing detailed conclusions from these calculations in which Schumann did not investigate the early, slow stages of growth, and assumed that all the water collected by the hailstone would freeze on its surface.

In the second part of his paper, Schumann realized that the growth rate of a hailstone will be limited by the rate at which it can dissipate

the latent heat of fusion. He therefore considered the heat balance of the hailstone and formulated relationships to determine, for given values of the ambient temperature, limiting values of the liquid-water content above which the water captured by the hailstone cannot all freeze and some may be shed in the form of droplets. His general treatment of the heat balance is sound although his theoretical estimates of the transfer coefficients for heat and water vapour are unreliable. Neeser-Scheer, Schumann was able to show that a hailstone might not be able to freeze all the water collected, even with air temperatures well below 0°C , and that at higher ambient temperatures, the growth of a hailstone might be even more restricted.

The validity of these conclusions was demonstrated by Ludlam (1950, 1952), who calculated the rate of heat transfer from a hailstone, using the empirical relations obtained by Kraemer (1946) for heat conduction and by Frisinger (1932) for mass transfer from spheres under forced convection. Subsequently, slightly different formulations of the problem have been given by List (1960, 1963), Machin (1963), List, Schnepp, and Method (1965), and List and Dussanith (1967), but Ludlam's original analysis is essentially correct for *smooth ice spheres* of $d < 3\text{ cm}$, and brings out clearly the physics of the problem. We shall now follow a slightly more general version of Ludlam's treatment and point out how this has to be modified to deal with large natural hailstones having rough, non-spherical surfaces.

Assuming that the hailstone grows by the accretion and freezing of water that spreads uniformly over its surface, the mass of water collected per unit time by a spherical stone of radius R , falling at velocity V relative to the cloud droplets, will be $E\pi R^2 V w$, where E is the average collection efficiency and w is the concentration of cloud water. If this is to be frozen completely, then the rate of release of heat is

$$\dot{Q}_i = E\pi R^2 V w [L_f + c_w(T_a - T_0) + c_i(T_0 - T_i)], \quad (6.19)$$

where T_0 is the melting temperature of ice, T_a the ambient air temperature, T_i the mean surface temperature of the hailstone, L_f the latent heat of fusion, and c_w , c_i are respectively the specific heats of water and ice.

The release of latent heat by the freezing of the accreted water causes the surface temperature of the hailstone to rise above that of its surroundings. The rate at which heat is transferred by conduction and forced convection from such a ventilated sphere is given by

$$\dot{Q}_s = 2\pi K E (T_i - T_a) N a, \quad (6.20)$$

updraught which, because of wind shear, is inclined to the vortical near the ground, so that particles falling from the high-level outflow may re-enter the updraught at low level and make a further ascent.

Particles arising on small embryos will grow into only small hail pellets on their first passage through the updraught and will be swept upward and forward along a trajectory such as *AA* in Fig. 6.31†, and

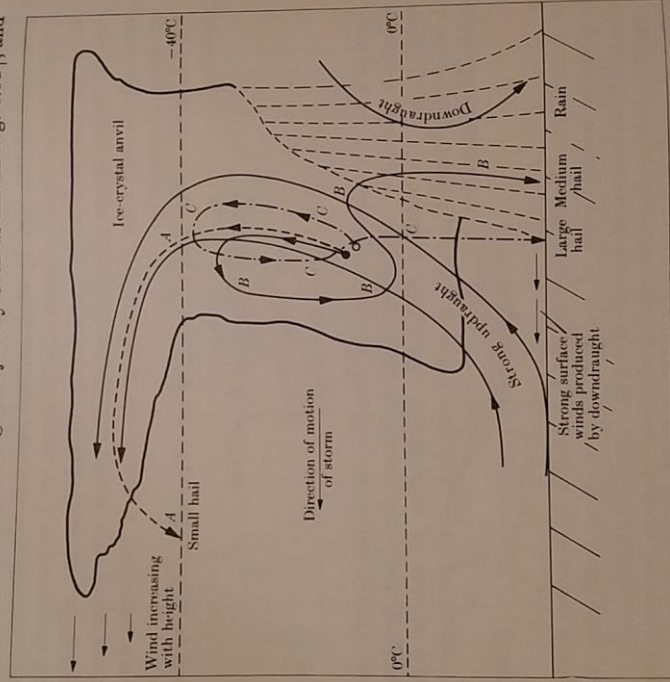


FIG. 6.31. Schematic representation of the structure of a hailstorm.

may fall, melt, and reach the ground as rain ahead, and to one side, of the core of the updraught. Rather larger particles may be thrown out of the core of the updraught but follow such paths as *BB*, grow on their downward journey, and re-enter the updraught at a lower level. If we now suppose that the updraught speed increases with height, then a small proportion of the re-entering particles that have a favourable size, may be lifted slowly by the updraught, growing at such a rate

† See also Fig. 8.34.

appreciably greater than this minimum value, the shrinkage during the fall to the ground is not very great.

However, these conclusions require modification for stones having a lobed structure and correspondingly higher values of the heat transfer coefficient. If, as the experiments of Bailey and Macklin (1969b) suggest, heat transfer from such stones of radius > 3 cm is three times that assumed by Ludlam, then eqn (6.39) should read

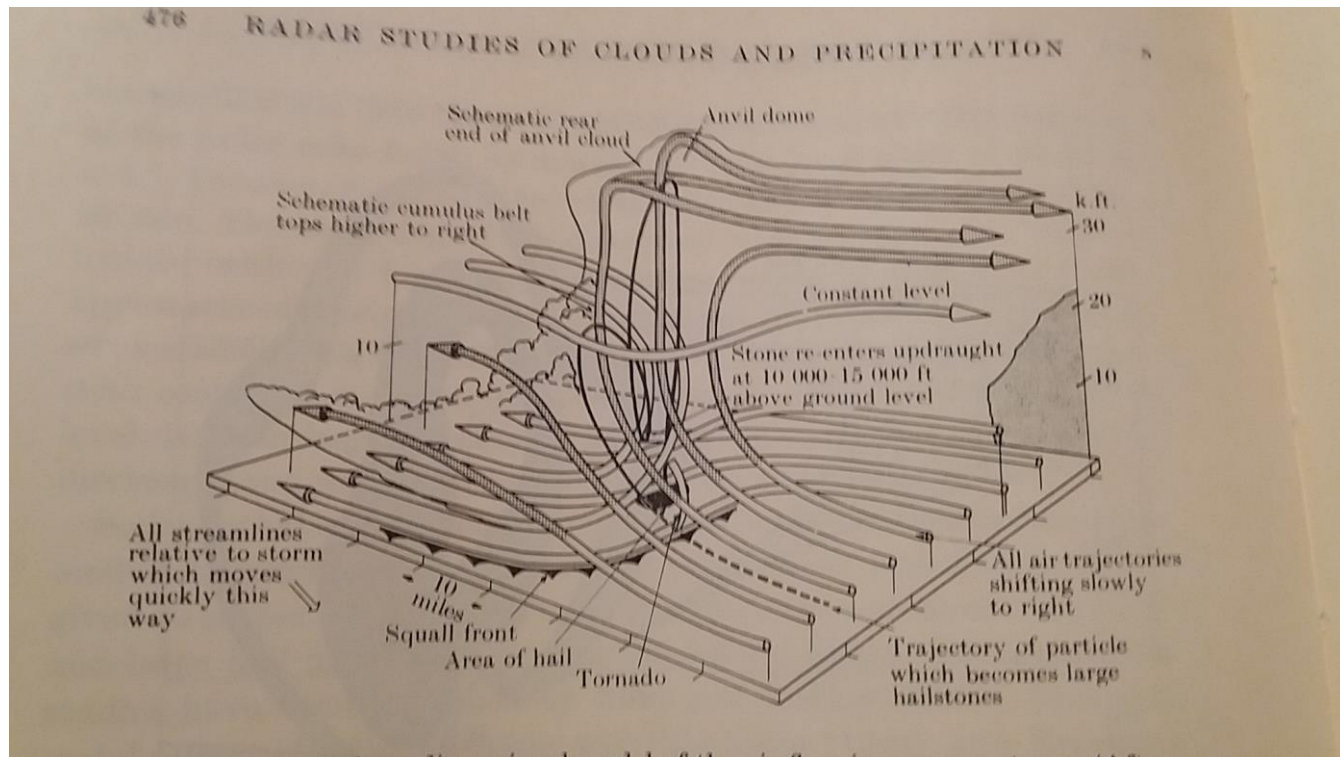
$$R_0^i - R_g^i = 1.53 \quad (6.40)$$

to give the following values:

R_g (cm)	3	4	6
R_0 (cm)	3.37	4.30	6.23
% change in R	12	7.5	3.8
% change in mass	41	24	12

which imply that even very large stones can undergo appreciable melting during their fall to the ground if they are lobed.

The rate of melting of small hailstones, spheres of solid or rime ice, in falling from the 0°C level to the ground, was computed by Mason (1956b). The calculations showed that, in a saturated atmosphere with a temperature gradient of $6.5^\circ\text{C}/\text{km}$, solid ice spheres of $R = 3$ mm would melt completely after falling 2.5 km, and soft hail pellets of the same size, but density 0.3 g cm^{-3} , after falling only 1 km from the 0°C level. Laboratory measurements by Drake and Mason (1966) on ice particles suspended in airstreams of controlled temperature, humidity, and velocity, gave melting times that agreed well with the calculated values especially if allowance was made in the latter for the heat content of the water retained by the particles during melting. The melting times of small ice cones of various shapes did not differ from those of spheres of equal volume by more than 10 per cent.



growth are also likely to be 1.2 to 3 times those computed previously. This implies that quite large stones may grow in liquid-water concentrations of a few g m^{-3} without becoming excessively wet or spongy. This is illustrated in Fig. 6.30 where Bailey and Macklin have plotted the effective critical liquid-water content, Ew_c , (because the collision efficiency E has not been accurately determined), as a function of the radius for both smooth ice spheres and for their artificial hailstones,

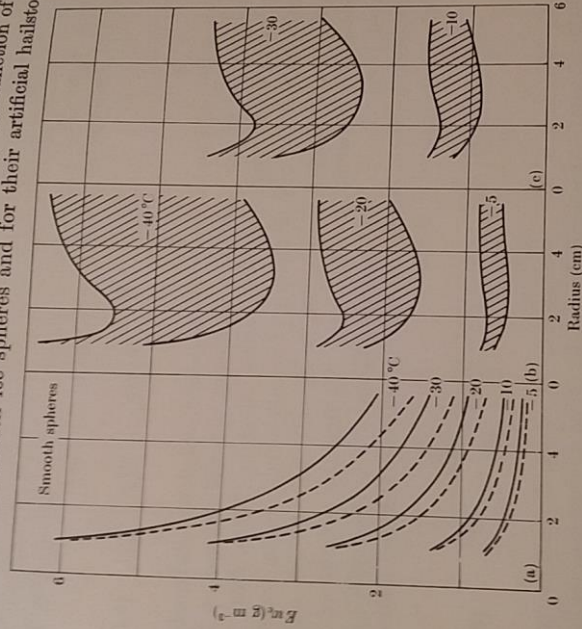


FIG. 6.30. Effective critical liquid water concentrations, Ew_c , as a function of hailstone radius: (a) shows the curves for smooth spheres, the full lines referring to the experimental data of Bailey and Macklin (1968b), and the dashed lines for the previously assumed value of $\alpha = 0.68$; (b) and (c) show the curves for hailstones, the two lines drawn for each value of ambient temperature indicating the spread in measured values of the transfer coefficients.

(From Bailey and Macklin (1968b).)

using both their experimentally-determined heat transfer coefficients and constant values of $\alpha = 0.68$.

The fraction, ϕ , of the deposited water that becomes frozen during wet growth may be calculated from eqn (6.24) with L_t replaced by ϕL_t , viz.

$$\phi = \frac{2\sqrt{2}}{Ew\mu^{\frac{1}{2}}}\left(\frac{\rho_a}{a}\right)^{\frac{3}{2}}\frac{\alpha\gamma}{L_t}\left(\frac{3C_D}{8\rho_l g}\right)^{\frac{1}{2}}\{L_v D\Delta\rho_v + K(T_s - T_a)\} - \frac{c_w(T_a - T_0)}{L_t} \quad (6.33)$$

isotopic composition of the ice in the various growth layers, to compute in consequence the trajectory of the stone relative to the updraught, and to relate these in a consistent manner to the crystal and air-bubble structure. The hailstone in question fell on 1 July 1968 near Cardiff, Wales, from a severe thunderstorm that produced some stones more than 7 cm in maximum dimension. A thin section, taken from near the centre of the stone, was photographed in transmitted and polarized light to provide Fig. 6.32(a) and (b). It is seen that, apart from the central core, the stone is divided into five main layers of alternatively clear and opaque ice. The first opaque layer is quite thin, while the second densely opaque layer has a clear layer running through it on one side of the stone. The lobe structure of the stone is also apparent. The crystal structure, shown in Fig. 6.32(b), coincides exactly with the bubble layers, the crystal sizes changing from a few millimetres to only a few tenths of millimetre as the ice goes from clear to opaque. Even in the thin clear layer embedded in the second opaque layer, there is a temporary reversion to relatively large crystals.

Small samples, about 20 mm³ in volume, were cut out of each layer along a number of radial directions, and their deuterium and ¹⁸O composition was determined by mass spectrometry using the technique described by Merlivat, Nief, and Roth (1965). Results of the deuterium analysis are shown in Fig. 6.33; the ¹⁸O analysis was very similar and showed the same variation of concentration with hailstone radius. Assuming that the hailstone grows in air making a saturated adiabatic ascent, the molecular species H₂¹⁶O, HD¹⁶O, and H₂¹⁸O are conserved, in the air but, during condensation, the cloud droplets become enriched, relative to the vapour, in the heavier isotopes. Because the droplets achieve very rapid equilibrium with the vapour phase, the molar fraction of deuterium (or ¹⁸O) in the drops is a function only of the vapour pressure and hence the temperature of the environment. (In fact, the quantity δD in Fig. 6.33, defined by

$$\delta D \simeq \left(\frac{N_L}{N_0} - 1 \right) \times 1000,$$

where N_L is the molar fraction of HDO molecules in the cloud droplets and N_0 that in water initially condensed at the cloud base, decreases with decreasing temperature.) Thus a measurement of the deuterium (or ¹⁸O) content in a melted sample of ice taken from a particular layer of a hailstone determines the temperature of the cloud droplets that

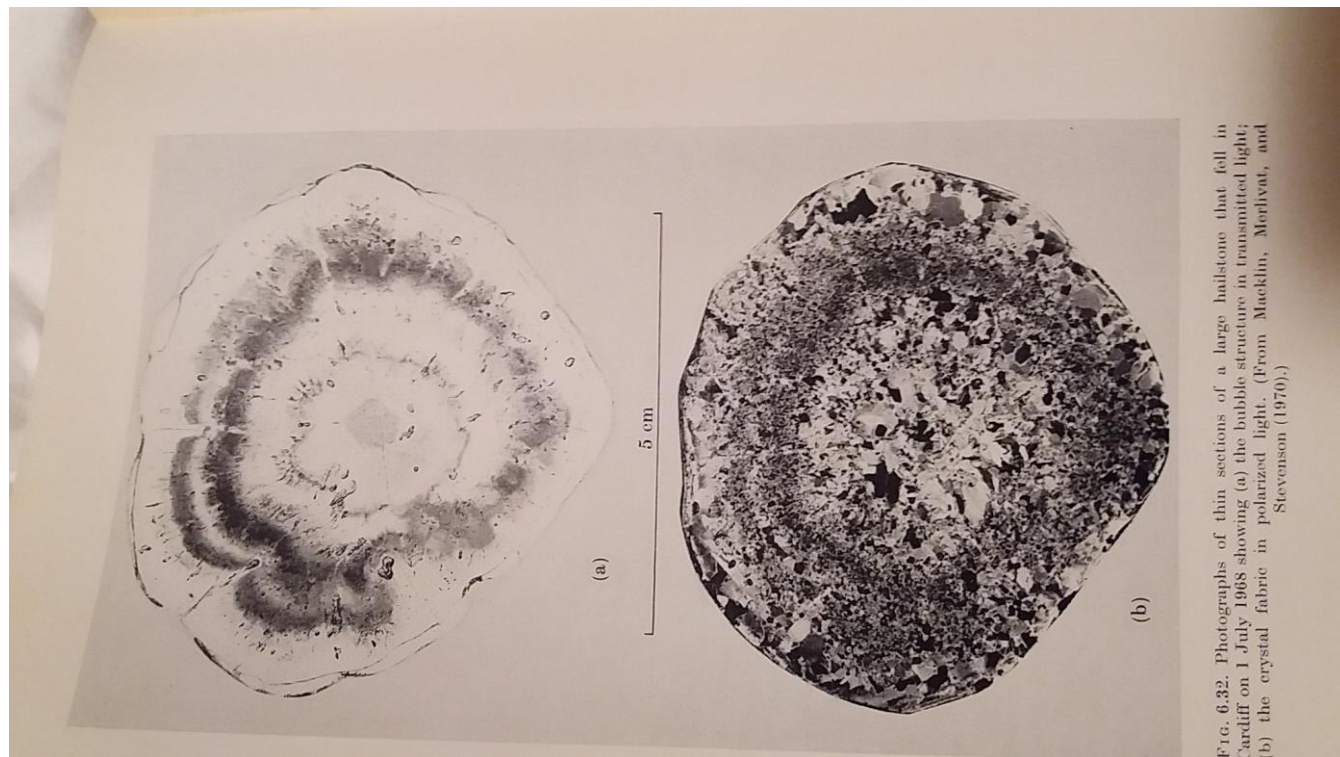


FIG. 6.32. Photographs of thin sections of a large hailstone that fell in Cardiff on 1 July 1968 showing (a) the bubble structure in transmitted light; (b) the crystal fabric in polarized light. (From Macklin, Morhval, and Stevenson (1970).)

were accreted to form this layer, this being also the temperature of the surrounding air at that time.

The data in Fig. 6.23 indicate the existence of two main growth zones where the deuterium concentration steadily changes with radius. The first begins at the onset of the first clear layer and the δD value decreases sharply with increasing radius until the first opaque layer is reached. Then there is an abrupt rise at the commencement of the second clear layer, with a subsequent fall to the thick outer opaque layer. There is also an indication of a final rise in the curve corresponding to the formation of the outermost clear layer. Using the curves of Fig. 6.33, curves showing how δD varies with temperature, and the temperature-height relationship for the adiabatic ascent, it is possible to determine how the height of the hailstone varied with its radius during growth, and also the air temperatures at these heights (see Fig. 6.34). This shows clearly the two main growth zones in the cloud, each of which gives rise to two growth layers in the stone. The layers are

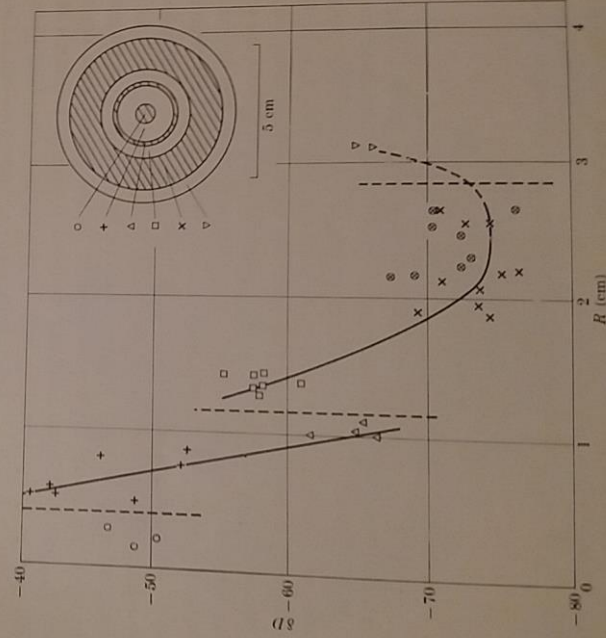


FIG. 6.33. The deuterium values as a function of hailstone radius. The equivalent spherical model is depicted in the upper right-hand corner. (From Macklin *et al.* (1970).)

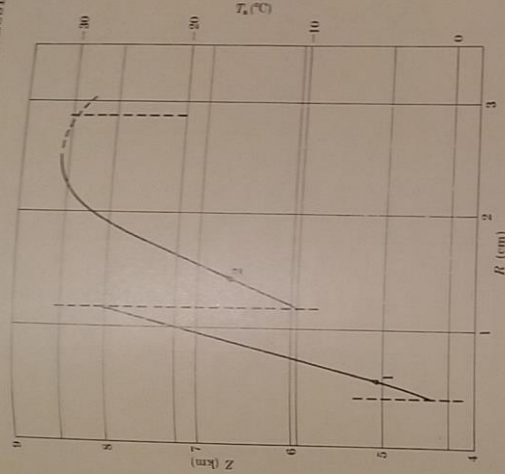


FIG. 6.34. The height of the hailstone as a function of its radius showing the two ascents. (From Maeklin *et al.* (1970).)

alternately clear and opaque, with concomitant changes in the crystal size from large to small. These events are readily interpreted in terms of the stone making a first ascent from near the 0°C level to about the $8\text{ km } (-25^{\circ}\text{C})$ level, where it was carried out of the updraught, fell down, and re-entered the updraught at a higher level ($6\text{ km } -10^{\circ}\text{C}$) than previously, and made a second ascent to the -30°C level. The flattening of this second part of the curve results from the fall-speed of the stone becoming comparable with the updraught. A spherical stone of this equivalent radius (2.3 cm), at this height, and having a drag coefficient of 0.55 , would have a fall-speed of 45 m s^{-1} .

The updraught profiles were calculated from eqn (6.9), viz.

$$(U - V) \frac{dR}{dz} = \frac{EVw}{4\rho_1}.$$

With values of dR/dz taken from Fig. 6.33, the terminal velocity of the stone calculated from $V = (8R\rho g/3C_D\rho_a)^{1/2}$, and w taken as the adiabatic value of the liquid-water concentration, the updraught speed U

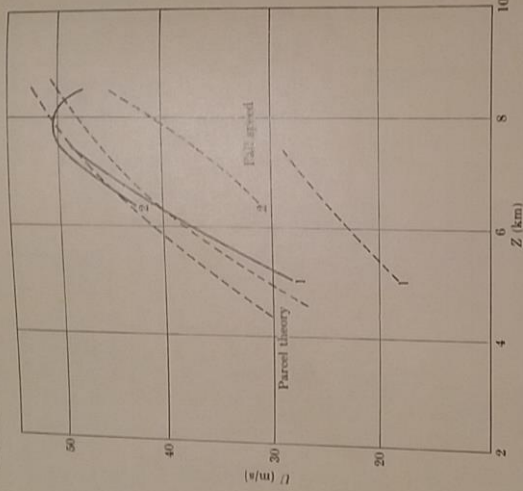


FIG. 6.35. Updraught profiles deduced for the two hailstone ascents. Included also are the updraughts deduced from the parcel method and assumed fall speeds of the stone at various heights. (From Macklin *et al.* (1970).)

was calculated as a function of height, and is shown in Fig. 6.35. Where they overlap, the profiles deduced from the first and second ascents are in good agreement, and both agree quite well with two theoretical computations based on variants of the parcel theory of adiabatic ascent.

6.6.8. The melting of hailstones

In their treatment of the melting of large hailstones, Ludlam (1958) and Macklin (1963) consider that most of the melt-water is shed so that, at any stage, the stone is covered with a film of water so thin that the surface temperature may be taken to be 0°C . The rate of melting of a smooth, spheroidal stone, falling in still, clear air, with its shortest axis vertical, may then be expressed by

$$L_t \frac{dm}{dt} = \frac{\pi R e^{\frac{1}{2}} A}{2a} \{L_v D \Delta \rho_v + K(T_s - T_a)\} = \frac{\pi R e^{\frac{1}{2}} A \beta}{2a}, \quad (6.37)$$

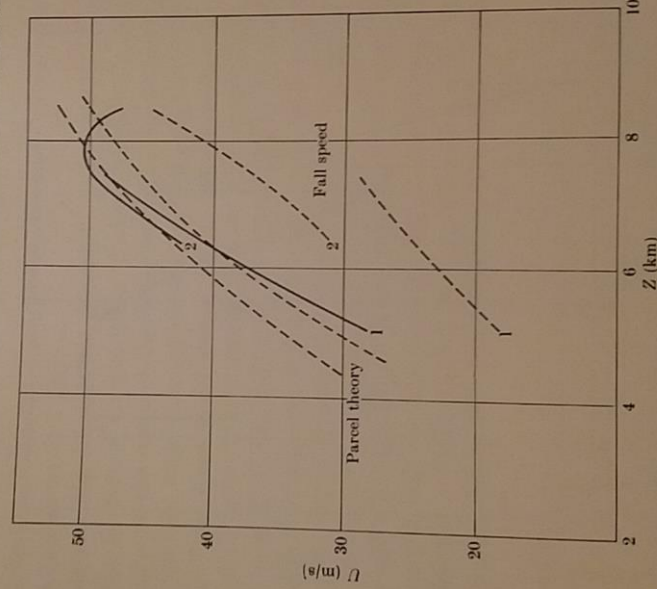


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where $\Delta\rho_s$ is now the difference between the vapour density in the environment and the saturation vapour density at 0°C . Integration of (6.37), with $m = \frac{4}{3}\pi\rho_s a^3\xi$, $\gamma = A/4\pi a^2$, $V^2 = 8\rho_s g a \xi / 3\rho_s C_D$,

$$\begin{aligned} da/dt &= V da/dz, & \rho_s &= 0.9 \text{ g cm}^{-3}, \\ \text{yields} & & & \\ a_0^3 - a_g^3 &= 2.23 \times 10^{-3} x \gamma C_D \frac{1}{\xi} \int_0^{\xi_0} \beta \rho_s^{\frac{1}{2}} \mu^{-\frac{1}{2}} dz, \end{aligned} \quad (6.38)$$

a_0, a_g being the major semi-axes of the stone at the 0°C level and at the ground respectively. The effect of the shape of the stone on its melting is therefore dependent on the term $x\gamma C_D \frac{1}{\xi}$, which is evaluated in Table 6.3. This shows that the change in the maximum dimension during melting is more pronounced the smaller the value of ξ . The fifth line of Table 6.3 shows the minimum size that the hailstone must possess at the 0°C level in order to just reach the ground after falling through still, clear air. In order to survive, the major axis of a spheroidal stone of $\xi = 0.4$ must be about twice the diameter of a spherical stone, but its mass would be only some 17 per cent larger.

For spherical stones falling through still, clear air in conditions appropriate to his Wokingham storm, Ludlam (1958) showed that eqn (6.38) takes the form

$$R_0^3 - R_g^3 = 0.51 \quad (6.39)$$

with solutions

$$\begin{array}{ccccc} R_0(\text{cm}) & 0 & 1 & 2 & 3 & 5 \\ R_g(\text{cm}) & 0.73 & 1.3 & 2.2 & 3.2 & 5.1 \end{array}$$

These figures show that a smooth, spherical hailstone must have a diameter rather larger than 1 cm at the 0°C level if it is to reach the ground in hot weather. On the other hand, if the size at this level is

TABLE 6.3
The influence of shape on the melting of smooth hailstones (after Macklin 1963)

ξ	0.4	0.6	0.8	1.0
C_D	0.73	0.67	0.61	0.55
x	0.80	0.76	0.72	0.68
$x\gamma C_D \frac{1}{\xi}$	1.62	0.99	0.75	0.60
a_0 (min)	1.50	0.94	0.80	0.70
mass spheroid	1.17	1.08	1.04	1.00
mass sphere				

appreciably greater than this minimum value, the shrinkage during the fall to the ground is not very great.

However, these conclusions require modification for stones having a lobed structure and correspondingly higher values of the heat transfer coefficient. If, as the experiments of Bailey and Macklin (1968*b*) suggest, heat transfer from such stones of radius >3 cm is three times that assumed by Ludlam, then eqn (6.39) should read

$$R_0^1 - R_g^1 = 1.53 \quad (6.40)$$

to give the following values:

R_g (cm)	3	4	6
R_0 (cm)	3.37	4.30	6.23
% change in R	12	7.5	3.8
% change in mass	41	24	12

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The rate of melting of small hailstones, spheres of solid or rime ice, in falling from the 0°C level to the ground, was computed by Mason (1956*b*). The calculations showed that, in a saturated atmosphere with a temperature gradient of $6.5^\circ\text{C}/\text{km}$, solid ice spheres of $R = 3$ mm would melt completely after falling 2.5 km, and soft hail pellets of the same size, but density 0.3 g cm^{-3} , after falling only 1 km from the 0°C level. Laboratory measurements by Drake and Mason (1966) on ice particles suspended in airstreams of controlled temperature, humidity, and velocity, gave melting times that agreed well with the calculated values especially if allowance was made in the latter for the heat content of the water retained by the particles during melting. The melting times of small ice cones of various shapes did not differ from those of spheres of equal volume by more than 10 per cent.